Name KEY Date 12 MC + 12 FR 24 pts total Gelatin Brand__________

AP Calculus Test 4.1-4.3, No calculator

Multiple Choice

B 1. $\int \left(x^2-2\right)^2 dx =$

(A) $\left(\frac{x^3}{3}-2x\right)^2+C$ (B) $\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$ (C) $\frac{\left(x^2-2\right)^3}{6x}+C$ (D) $\frac{2x}{3}\left(x^2-2\right)^3+C$ (E) $\left(\frac{x^2-2}{3}\right)^3+C$

$\int (x^4-4x^2+4)\,dx$

$\frac{1}{5}x^5-\frac{4}{3}x^3+4x+C$

A 2. If $f'(x)=\frac{x+1}{\sqrt{x}}$ and $f(1)=0$, then $f(4)=$

(A) $\frac{20}{3}$ (B) $\frac{4}{3}$ (C) $-\frac{4}{3}$ (D) $-\frac{8}{3}$ (E) $\frac{3}{4}$

$f'(x)=\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}$

$f'(x)=x^{1/2}+x^{-1/2}$

$f(x)=\frac{2}{3}x^{3/2}+2x^{1/2}+C$

@ (1,0): $0=\frac{2}{3}+2+C$

$0=\frac{8}{3}+C$

$C=-\frac{8}{3}$

So, $f(x)=\frac{2}{3}x^{3/2}+2x^{1/2}-\frac{8}{3}$

So, $f(4)=\frac{2}{3}(\sqrt{4})^3+2(\sqrt{4})-\frac{8}{3}$

$=\frac{2}{3}(8)+4-\frac{8}{3}$

$f(4)=\frac{2}{3}(8)+4-\frac{8}{3}$

$=\frac{16}{3}-\frac{8}{3}+4$

$=\frac{8}{3}+4$

$f(4)=\frac{20}{3}$

C 3. $\int \frac{\sin 2x}{\cos x}dx =$

(A) $\cos x+C$ (B) $2\cos x+C$ (C) $-2\cos x+C$ (D) $-\cos 2x+C$ (E) $\cos 2x+C$

* Double angle ID: $\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x-\sin^2 x$

$\int \frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}dx$

$2\int \sin x\,dx$

$2(-\cos x)+C$

$-2\cos x+C$

B 4. $\int \csc x(\cot x+\sin x)\,dx =$

(A) $-\csc x+C$ (B) $-\csc x+x+C$ (C) $\sec x+\cos x+C$ (D) $\csc x+x+C$ (E) $-\sec x+\tan x+C$

$\int (\csc x\cot x+\csc x\cdot\sin x)\,dx$ * $\csc x=\frac{1}{\sin x}$

$\int (\csc x\cdot\cot x+1)\,dx$

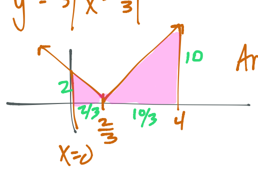
$-\csc x+x+C$

D 5. $\int_0^4 |3x-2|\,dx =$

(A) $\frac{33}{2}$ (B) $\frac{50}{3}$ (C) $\frac{35}{2}$ (D) $\frac{52}{3}$ (E) $\frac{47}{3}$

$y=|3x-2|$

$y=3\left|x-\frac{2}{3}\right|$



Area $=\int_0^1 |3x-2|\,dx = \frac{1}{2}\left(\frac{2}{3}\right)(2)+\frac{1}{2}\left(\frac{10}{3}\right)(10)$

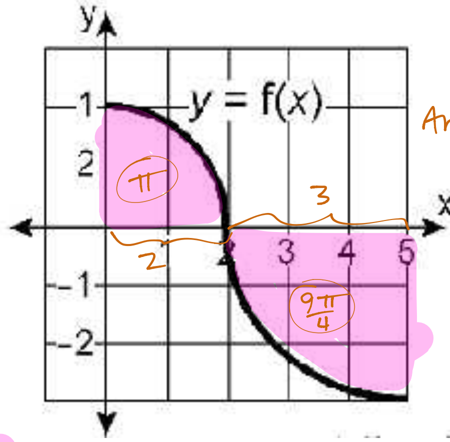
$=\frac{2}{3}+\frac{50}{3}$

$=\frac{52}{3}$

B

6. If the graph of the function $f(x)$ below is composed of two quarter circles, then $\int_5^0 f(x)\,dx =$

Right to Left!!



Areas: small $= \left(\frac{1}{4}\right)\pi(2^2) = \pi$

big $= \left(\frac{1}{4}\right)\pi(3^2) = \frac{9\pi}{4}$

$\int_5^0 f(x)\,dx$

$= -\int_0^5 f(x)\,dx$

$= -\left[\pi - \frac{9\pi}{4}\right]$

$= -\left[\frac{4\pi}{4} - \frac{9\pi}{4}\right]$

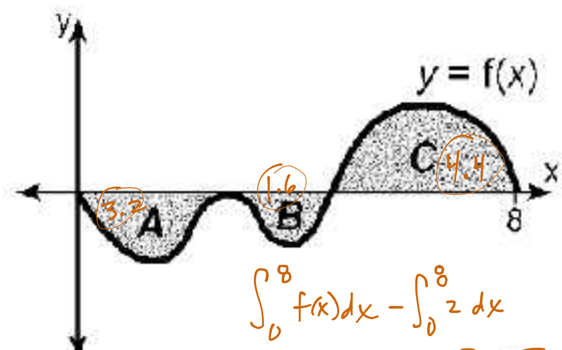
$= -\left[-\frac{5\pi}{4}\right]$

$= \frac{5\pi}{4}$

(A) $\frac{13\pi}{4}$ (B) $\frac{5\pi}{4}$ (C) $-\frac{5\pi}{4}$ (D) $\frac{13\pi}{2}$ (E) 13π

D

7. In the graph below, the areas of regions A, B, and C are $A = 3.2$, $B = 1.6$, and $C = 4.4$.



$\int_0^8 f(x)\,dx - \int_0^8 2\,dx$

$= [-3.2 - 1.6 + 4.4] - [2(8-0)]$

$= (-4.8 + 4.4) - 16 = -0.4 - 16 = -16.4$

What is the value of $\int_0^8 (f(x) - 2)\,dx$?

(A) 16.4 (B) −0.4 (C) −15.6 (D) −16.4 (E) −2.4

C

8. $\int_0^{\pi/4} 4\sec^2 x\,dx =$

(A) 4π (B) 0 (C) 4 (D) π (E) 8

$= 4\int_0^{\pi/4} \sec^2 x\,dx$

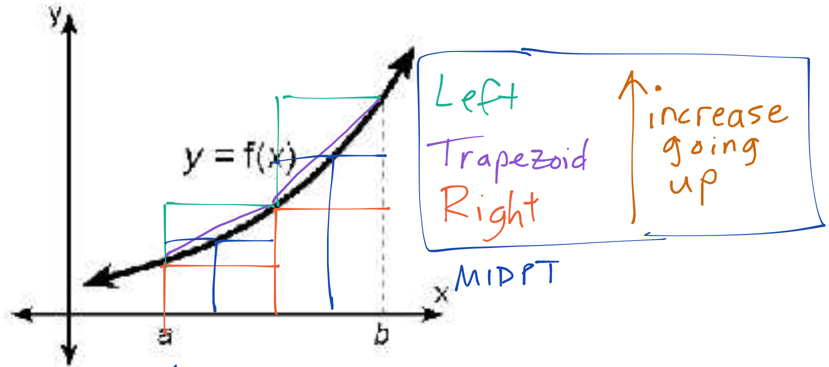
$= 4\tan x \Big|_0^{\pi/4}$

$= 4\left[\tan\frac{\pi}{4} - \tan 0\right]$

$= 4[1 - 0]$

$= 4$

C 9. According to the graph below, which of the following is **false** for the function $f(x)$ when the indicated Riemann & Trapezoidal sums are used to approximate the value of $\int_a^b f(x)\,dx$?



(A) Right hand sum $\geq$ Midpoint sum ✓ True for sure
(B) Midpoint sum $\leq$ Trapezoidal sum ? hard to tell
(C) Left hand sum $\geq$ Trapezoidal sum FALSE for sure!!
(D) Left hand sum $\leq$ Right hand sum ✓ True for sure
(E) Trapezoidal sum $\leq$ Right hand sum ✓ True for sure

C 10. The table below gives various values of a continuous function $f(x)$ on the closed interval $[0,8]$.

x	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
f(x)	0.8	1.2	3.1	0.6	0.4	2.2	3.0	2.4	3.6

Using the given values and four subintervals of width 2, the midpoint Riemann approximation of $\int_0^8 f(x)\,dx$ is

(A) 12.4 (B) 11.8 (C) 12.8 (D) 12.6 (E) 13.2

$\approx M_4 = 2[1.2 + 0.6 + 2.2 + 2.4] = 2[6.4] = 12.8$

D 11. The table of values below represents a continuous function $g(x)$.

Not used

x	1	3	4	7	~~9~~
$g(x)$	20	40	60	50	~~70~~

Using 3 subintervals, what is the trapezoidal approximation of $\int_1^7 g(x)\,dx =$

(A) 135 (B) 305 (C) 270 (D) 275 (E) 290

$\int_1^7 g(x)\,dx \approx T_3 = \frac{1}{2}\left[2(20+40) + 1(40+60) + 3(60+50)\right]$
$= \frac{1}{2}[120 + 100 + 330] = 60 + 50 + 165$
$= 110 + 165 = 275$

A 12. If $\int_{-2}^{3} f(x)\,dx = 5$, $\int_6^3 f(x)\,dx = -4$, and $\int_6^5 f(x)\,dx = 2$, what is $\int_5^{-2} f(x)\,dx$?

(A) -11 (B) -7 (C) -1 (D) 1 (E) 3

$\int_5^{-2} f(x)\,dx = \int_5^6 f(x)\,dx + \int_6^3 f(x)\,dx + \int_3^{-2} f(x)\,dx$

$= (-2) + (-4) + (-5) = -11$

Short Answer: Evaluate the following indefinite integrals. Remember, rewriting is the key, and don't forget your $+C$.

13. $\int\left(5^x+\frac{4-x}{x}\right)dx=$

$\int(5^x+\frac{4}{x}-1)dx$

$\frac{1}{\ln 5}\cdot 5^x + 4\ln|x| - x + C$ √1 √2 √3

$\frac{d}{dx}5^x = 5^x(\ln 5)$

$\boxed{+C \text{ is 1 check for all}}$

14. $\int\left(\frac{\sqrt{1-t^2}\cdot\sqrt[3]{t^2}+1}{\sqrt{1-t^2}}\right)dt=$

√10 split up

$\int\left(\frac{\cancel{\sqrt{1-t^2}}\cdot\sqrt[3]{t^2}}{\cancel{\sqrt{1-t^2}}}+\frac{1}{\sqrt{1-t^2}}\right)dt$

$\int\left(t^{2/3}+\frac{1}{\sqrt{1-t^2}}\right)dt$

$\frac{3}{5}t^{5/3} + \arcsin t + C$

or

$\frac{3}{5}t^{5/3} + \sin^{-1}t + C$

√11 √12

15. $\int 4\sqrt{m}\,(2m-5)^2\,dm=$

$4\int\left(m^{1/2}(4m^2-10m+25)\right)dm$ √4 Xpand

$4\int\left(4m^{5/2}-10m^{3/2}+25m^{1/2}\right)dm$

$4\left[4\cdot\left(\frac{2}{7}\right)m^{7/2}-10\left(\frac{2}{5}\right)m^{5/2}+25\left(\frac{2}{3}\right)m^{3/2}\right]+C$

√5 variables

$4\left[\frac{8}{7}m^{7/2}-4m^{5/2}+\frac{50}{3}m^{3/2}\right]+C$

$\frac{32}{7}m^{7/2}-16m^{5/2}+\frac{200}{3}m^{3/2}+C$

16. $\int\left(\cot^2 x-\sec^2 x\right)dx=$

√6

$\int(\csc^2 x-1-\sec^2 x)dx$

$-\cot x - x - \tan x + C$

√8 √7

$+C$ √9

must be attached to an expression, not JUST $+C$

12 F.R. points