Name____ Date____________________ Period__________

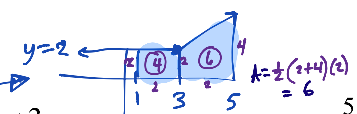
AP Calculus Test 4.1-4.3, No calculator

Multiple Choice

A 1. $\int \sec x \tan x dx =$ (A) $\sec x + C$ (B) $\tan x + C$ (C) $\frac{\sec^2 x}{2} + C$ (D) $\frac{\tan^2 x}{2} + C$ (E) $\frac{\sec^2 x \tan^2 x}{2} + C$

since $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\int \sec x \tan x \, dx = \sec x + C$



D 2. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3 \end{cases}$. What is the value of $\int_1^5 f(x)dx$?

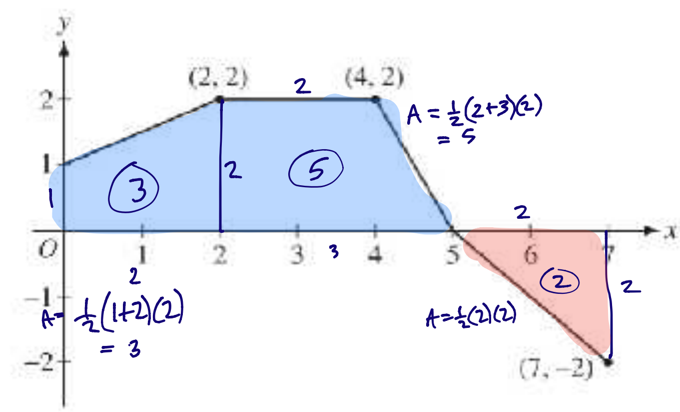
$= 4 + 6$

$= 10$

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

A 3. The graph of a function f is shown at right. What is the value of $\int_0^7 f(x)dx$? $= 3 + 5 - 2 = 6$

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18



Graph of f

x	0	2	4	6
$f(x)$	4	k	8	12

D 4. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

$\int_0^6 f(x)dx \approx T_3 = \left(\frac{1}{2}\right)(2)[4 + 2k + 2(8) + 12] = 52$

$32 + 2k = 52$

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

$2k = 20$

$k = 10$

B 5. $\int (x^3 + 1)^2 dx =$ (A) $\frac{1}{7}x^7 + x + C$ (B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$ (C) $6x^2(x^3 + 1) + C$ (D) $\frac{1}{3}(x^3 + 1)^3 + C$ (E) $\frac{(x^3 + 1)^3}{9x^2} + C$

$\int (x^6 + 2x^3 + 1)dx$

$\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

C 6. $\int_{1}^{4} |x-3|\,dx =$ $\frac{1}{2}(2)(2)+\frac{1}{2}(1)(1)$ (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

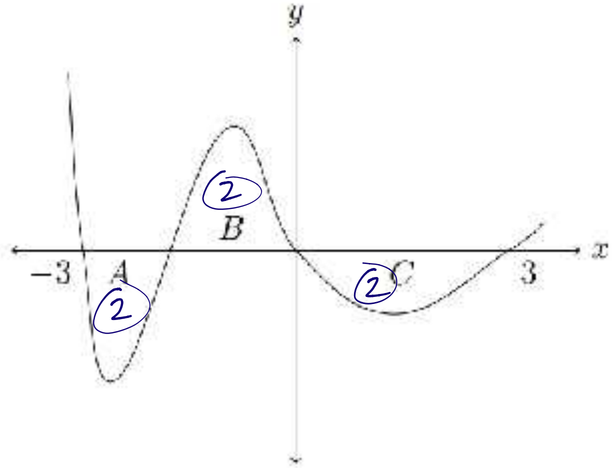
$= 2+\frac{1}{2}$

$= \frac{5}{2} = 2.5$

1 3 4

D 7. The regions A, B, and C in the figure at right are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x)+1)\,dx$?

(A) -2 (B) -1 (C) 2 (D) 4 (E) 7



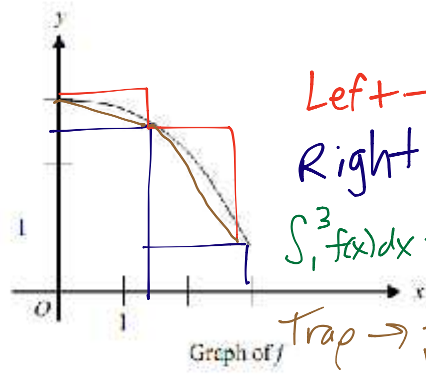
$\int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$

$(-2+2-2) + (1)(3-(-3))$

$-2 + 6$

4

C 8. The graph of the function f is shown below for $0 \le x \le 3$. Of the following, which has the least value?



Left → too big

Right → too small

$\int_1^3 f(x)\,dx$ → exact

Trap → too small but bigger than left

(A) $\int_{1}^{3} f(x)\,dx$

(B) Left Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

B 9. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx$

$-17 + 4$

−13

B 10. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c \left(f(x) - g(x)\right)dx =$

(A) $P-Q+R-S$ (B) $P-Q-R+S$ (C) $P-Q-R-S$ (D) $P+Q-R-S$ (E) $P+Q-R+S$

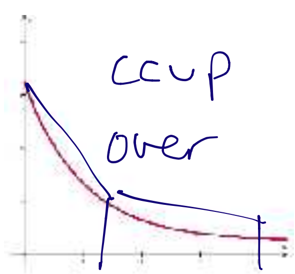
$\int_a^b f\,dx + \int_b^c f(x)\,dx - \int_a^b g(x)\,dx - \int_b^c g(x)\,dx$

$P - Q - (R) - (-S)$

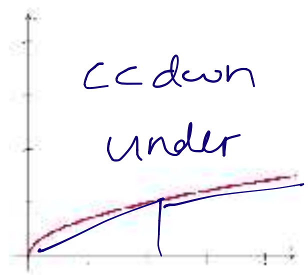
$P - Q - R + S$

A 11. If a trapezoidal sum over-approximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

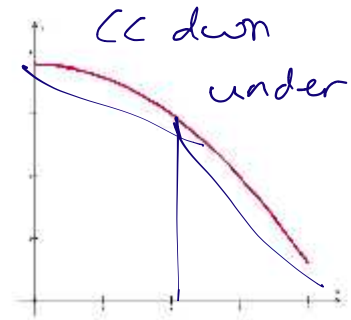
(A)



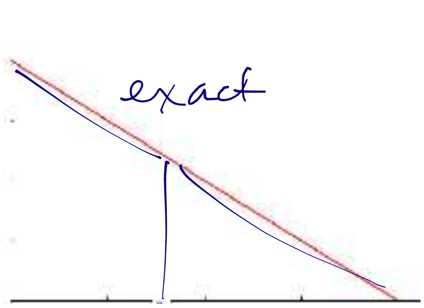
(B)



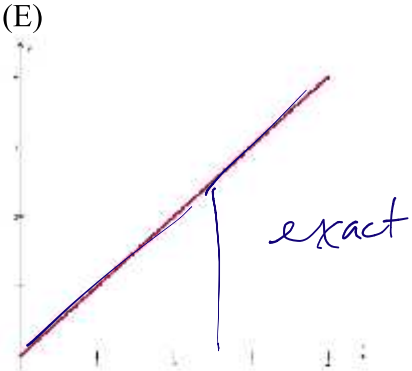
(C)



(D)



(E)



B 12. The function f is continuous on the closed interval $[2,13]$ and has values as shown in the table below. Using the intervals $[2,3]$, $[3,5]$, $[5,8]$, and $[8,13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

1 2 3 5

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50

$$\int_2^{13} f(x)\,dx \approx L_4 = 1(6) + 2(-2) + 3(-1) + 5(3)$$
$$= 6 - 4 - 3 + 15$$
$$= 14$$

B 13. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 [f(x)+g(x)]dx =$ $\int_3^5 (g(x)+7+g(x))dx = 2\int_3^5 g(x)\,dx + \int_3^5 7\,dx$

$2\int_3^5 g(x)\,dx + 7(2)$

$2\int_3^5 g(x)\,dx + 14$

(A) $2\int_3^5 g(x)\,dx+7$ (B) $2\int_3^5 g(x)\,dx+14$ (C) $2\int_3^5 g(x)\,dx+28$ (D) $\int_3^5 g(x)\,dx+7$ (E) $\int_3^5 g(x)\,dx+14$

D 14. The function *f* is continuous on the closed interval $[2,14]$ and has values as show in the table below. Using three subintervals indicated by the data, what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

3 5 4

x	2	5	10	14
$f(x)$	12	28	34	30

$\int_2^{14} f(x)dx \approx R_3 = 3(28)+5(34)+4(30)$
$= 84+170+120$
$= 204+170$
$= 374$

(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

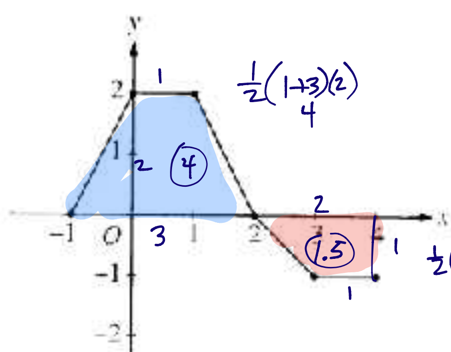
C 15. The most general antiderivative of $f(x)=(\sec x)\left(\frac{\cot x}{\sin x}\right)$ is $\int (\sec x)\left(\frac{\cot x}{\sin x}\right)dx = \int \left(\frac{1}{\cancel{\cos x}}\right)\left(\frac{\cancel{\cos x}}{\sin x}\right)\left(\frac{1}{\sin x}\right)dx = \int \csc^2 x\,dx$
$-\cot x + C$

(A) $\sec x\tan x+C$ (B) $-\csc x\cot x+C$ (C) $-\cot x+C$ (D) $\cos x+C$

E 16. If $\int_{-1}^{3} f(x)dx = 2$ and $\int_{2}^{3} f(x)dx = -1$, find $\int_{-1}^{2}[2f(x)]dx$ $= 2\int_{-1}^{2} f(x)dx = 2\left[\int_{-1}^{3} f(x)dx + \int_{3}^{2} f(x)dx\right]$

(A) 2 (B) -3 (C) 3 (D) -6 (E) 6

$= 2[2+1]$
$= 2(3)$
$= 6$



$\frac{1}{2}(1+3)(2)$
4

$\frac{1}{2}(2+1)(1)$
$\frac{3}{2}$

$\int_{-1}^{4} f(x)dx = 4 - 1.5 = 2.5$

B 17. The graph of a piecewise-linear function *f*, for $-1\le x\le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)dx$? (A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

A 18. If f is continuous for all x, which of the following integrals ~~necessarily~~ must have the same value?

I. $\int_a^b f(x)dx$ Basis for Comparision

II. $\int_{0}^{b-a} f(x+a)dx$ left a; $a-a=0$; left a

III. $\int_{a+c}^{b+c} f(x+c)dx$ right c; left c; Wrong directions

(A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None

left a

Short Answer: Evaluate the following indefinite integrals. Remember, rewriting is the key, and don't forget your $+C$.
Evaluate 4 of 6 of the following integrals (or get them all correct for amazing bonus points).

12. $\int e \csc x \tan^2 x dx$

$e\int \frac{1}{\cancel{\sin x}}\left(\frac{\cancel{\sin x}}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx$

$e\int \left(\frac{1}{\cos x}\cdot\frac{\sin x}{\tan x}\right)dx$

$e\int \sec x \cdot \tan x \, dx$

$e \cdot \sec x + C$

13. $\int \frac{2}{5\cdot 7^{-x}} dx$

$\frac{2}{5}\int 7^{x} dx$

$\frac{2}{5}\cdot\frac{1}{\ln 7}\cdot 7^{x} + C$

correction

14. $\int\left(\frac{4x+3\sqrt[3]{x}-x^2}{2x}\right)dx$

$\int\left(\frac{4x}{2x}+\frac{3x^{1/3}}{2x}-\frac{x^2}{2x}\right)dx$

$\int\left(2+\frac{3}{2}x^{-2/3}-\frac{1}{2}x\right)dx$

$2x+\frac{9}{2}x^{1/3}-\frac{1}{4}x^2+C$

15. $\int 2\sqrt{x}(3x-2)^2 dx$

$\int\left(2x^{1/2}(9x^2-12x+4)\right)dx$

$\int\left[18x^{5/2}-24x^{3/2}+8x^{1/2}\right]dx$

$\frac{36}{7}x^{7/2}-\frac{48}{5}x^{5/2}+\frac{16}{3}x^{3/2}+C$

16. $\int\left(\frac{4}{\pi x}-\frac{2}{\sin^2 x}\right)dx$

$\int\left[\frac{4}{\pi}\left(\frac{1}{x}\right)-2\csc^2 x\right]dx$

$\frac{4}{\pi}\ln|x|+2\cot x+C$

17. $\int\left(\frac{e^{-x}-1}{e^{-x}}\right)dx$

$\int\left(\frac{e^{-x}}{e^{-x}}-\frac{1}{e^{-x}}\right)dx$

$\int(1-e^{x})dx$

$x-e^{x}+C$