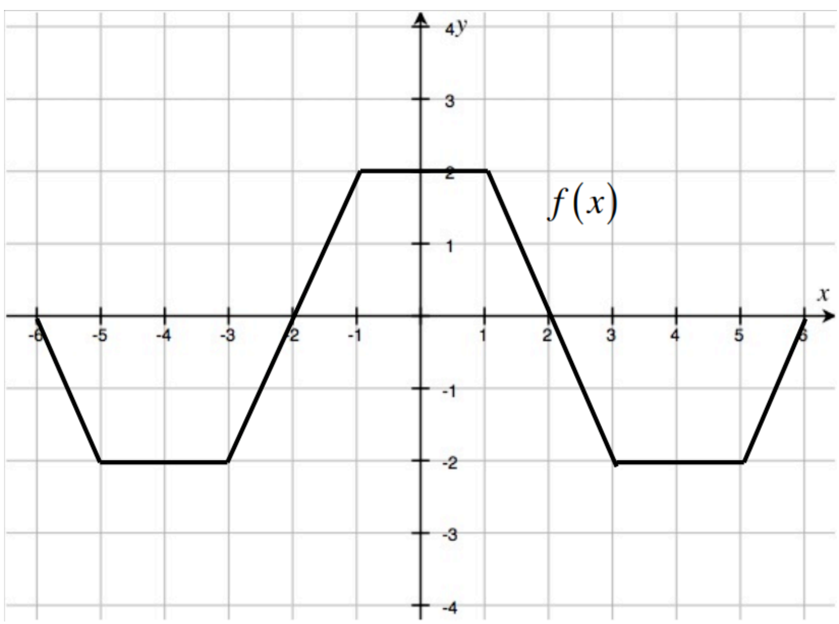
Name____ Date_______________ Dance Move__________

TEST: AP Calculus: Test—3.6-4.2. CALCULATOR PERMITTED

PART I: Multiple Choice. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

_____ 1. Freudian Pizza Parlor sells a soda for \$1.40 and a slice of Freudian pizza for \$2.50. In any given week, they sell 500 sodas and 1,000 slices of pizza. The proprietors of the parlor determine that for every dime they increase the price of a pizza slice, they will sell 10 fewer sodas and 20 fewer slices. At what price should they sell their pizza slice if they wish to maximize their revenue?

(A) \$4.80 (B) \$3.60 (C) \$3.40 (D) \$3.00 (E) \$2.75



_____ 2. The graph of $f(x)$ is shown above. Which of the following must be true?

I. $\int_{-6}^{-2} f(x)\,dx = \int_{2}^{6} f(x)\,dx$

II. $\int_{-2}^{2} f(x)\,dx = \int_{6}^{2} f(x)\,dx$

III. $\int_{0}^{1} f(x)\,dx = \int_{-2}^{-6} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

_____ 3. If $\int_{-1}^{5} g(x)\,dx = 11$ and $\int_{5}^{1} g(x)\,dx = -8$, what is $\int_{-1}^{1} g(x)\,dx$?

(A) -6 (B) 19 (C) -19 (D) 3 (E) -3

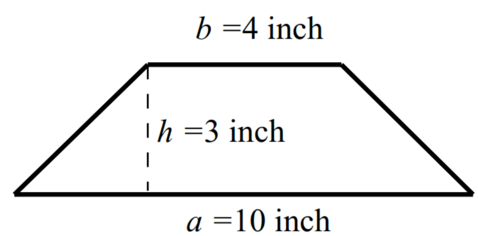
_____ 4. Estimations for $\int_{0}^{3} \left(30 - x^3\right) dx$ are calculated using a left Riemann sum (*L*), a right Riemann sum (*R*), and using trapezoids (*T*), each using 4 subintervals of equal width. Which of the following lists the estimations from least to greatest?

(A) $R < T < L$ (B) $R < L < T$ (C) $T < L < R$ (D) $L < R < T$ (E) $L < T < R$

_____ 5. The function $h(x)$ is continuous on the interval $[-4, 12]$. Selected values of x and $f(x)$ are given in the table below. If $\int_{-4}^{12} f(x)\,dx$ is estimated using a right Riemann sum with 4 equal subintervals, a left Riemann sum with 4 equal subintervals, trapezoids with 4 equal subintervals, and a midpoint Riemann sum with 2 equal subintervals, what is the difference between the largest and smallest estimation?

x	-4	0	4	8	12
$f(x)$	3	9	-2	-6	-3

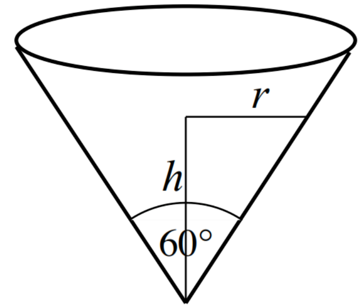
(A) 68 (B) 32 (C) 24 (D) 16 (E) 8



_____ 6. A trapezoid is pictured above. It has a base, *a*, that is a constant 10 inches while its top base, *b*, is increasing at a rate of 3 inches per minute while its height, *h*, is decreasing at a rate of $\frac{1}{2}$ inches per minute. When the top base is 4 inches and the height is 3 inches, how fast, in square inches per minute, is the area of the trapezoid changing?

(A) 8 (B) 2 (C) 1 (D) $-\frac{3}{4}$ (E) $-\frac{5}{2}$

_____ 7. Kool-Aid is draining from a conical tank whose base angle is $60°$ as shown in the figure at the right. When the height of the Kool-Aid is 3 feet, its height is decreasing at 6 inches per hour. At this moment, how fast, in cubic feet per hour, is the volume of the Kool-Aid decreasing?



(A) 162π (B) 18π (C) $\frac{13\pi}{2}$ (D) $\frac{3\pi}{2}$ (E) 3π

_____ 8. $\int \frac{\pi}{x^e}\,dx =$

(A) $\frac{\pi x^{1-e}}{1-e}+C$ (B) $\frac{\pi}{(e+1)x^{e+1}}+C$ (C) $\frac{\pi}{x^{e+1}}+C$ (D) $\pi x^{1-e}+C$ (D) $\frac{\pi x^{e+1}}{e+1}+C$

_____ 9. Use a tangent line approximation for $g(x)=\sqrt{x}$ at $x=64$ to estimate $\sqrt{65}-\sqrt{63}$.

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{16}$ (D) $\frac{1}{32}$ (E) 0

_____ 10. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx =$

(A) $\frac{\left(\sqrt{x}-1\right)^3}{3\sqrt{x}}+C$ (B) $\frac{\left(\sqrt{x}-1\right)^3}{3}+C$ (C) $\frac{2}{3}x^{3/2}+2x^{1/2}+C$

(D) $\frac{1}{2}x^{1/2}-\frac{4}{3}x+x^{1/2}+C$ (E) $\frac{2}{3}x^{3/2}-2x+2x^{1/2}+C$

PART II: Free Response—Use Proper Notation

11. I was out collecting data yesterday and used it to approximate a **differentiable** function $y = f(x)$ represented in the table below.

x	0	4	8	11	14	15	16
y	30	6	1	2	0	-1	0

. . . use my data to **approximate** $\int_0^{16} f(x)dx$ using the following methods using the given number of subintervals, n. (**simplify your answers**):

(a) Left end-point Riemann Sums ($n = 6$).

(b) Right end-point Riemann Sums ($n = 6$)

(c) Midpoint Riemann Sums ($n = 3$)

(d) Trapezoidal Rule ($n = 6$)

(e) Can any of the above calculations represent the approximate area under the function $y = f(x)$ on $[0,16]$? Why or why not?

(f) **Approximate** $f'(12)$ from the table of values. Make sure to show your difference quotient.

(g) If the **secant** line on the interval $[11,14]$ was used to approximate $f(12)$, given that $f'(x) < 0$ and $f''(x) < 0$ for all $x \in (11,14)$, would this approximation of $f(12)$ be an over or under approximation? Explain why..