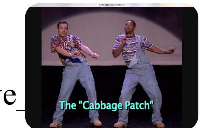
Name KEY Date Friday, 1/22/2016 Dance Move 

TEST: AP Calculus: Test—3.6-4.2. CALCULATOR PERMITTED 20 checks

PART I: Multiple Choice. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

C 1. Freudian Pizza Parlor sells a soda for \$1.40 and a slice of Freudian pizza for \$2.50. In any given week, they sell 500 sodas and 1,000 slices of pizza. The proprietors of the parlor determine that for every dime they increase the price of a pizza slice, they will sell 10 fewer sodas and 20 fewer slices. At what price should they sell their pizza slice if they wish to maximize their revenue?

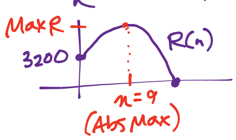
(A) \$4.80 (B) \$3.60 (C) \$3.40 (D) \$3.00 (E) \$2.75

Let n = # of Dime increases to pizza price

$R = (1.40)(500-10n) + (2.50+0.10n)(1000-20n)$

$R = 700 - 14n + 2500 - 50n + 100n - 2n^2$

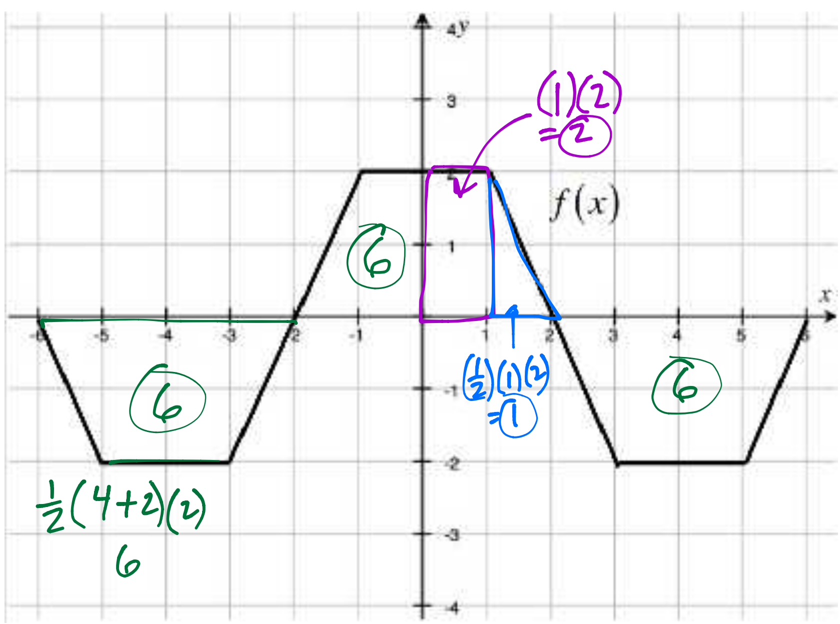
$R = -2n^2 + 36n + 3200$



$R'(n) = -4n + 36 = 0$

$n = 9$

* so they should raise the price of a pizza slice 9 dimes, or 90¢, from \$2.50/slice to \$3.40/slice



D 2. The graph of $f(x)$ is shown above. Which of the following must be true?

I. $\int_{-6}^{-2} f(x)\,dx = \int_{2}^{6} f(x)\,dx$ $-6 = -6$ ✓

II. $\int_{-2}^{2} f(x)\,dx = \int_{6}^{2} f(x)\,dx$ $6 = 6$ ✓

III. $\int_{0}^{1} f(x)\,dx = \int_{-2}^{-6} f(x)\,dx + \int_{1}^{2} f(x)\,dx$ $2 = 6 + 1$, $2 \neq 7$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

D 3. If $\int_{-1}^{5} g(x)\,dx = 11$ and $\int_{5}^{1} g(x)\,dx = -8$, what is $\int_{-1}^{1} g(x)\,dx$?

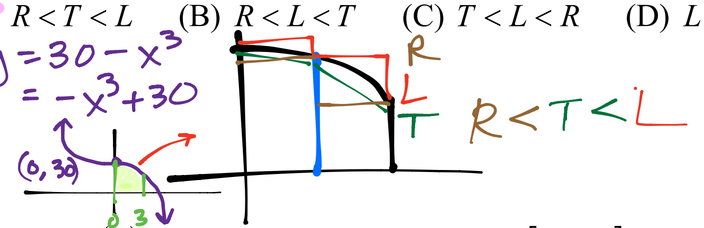
(A) -6 (B) 19 (C) -19 (D) 3 (E) -3

$$\int_{-1}^{1} g(x)dx = \int_{-1}^{5} g(x)dx + \int_{5}^{1} g(x)dx = 11 + -8 = 3$$

A 4. Estimations for $\int_{0}^{3} \left(30 - x^3\right) dx$ are calculated using a left Riemann sum (L), a right Riemann sum (R), and using trapezoids (T), each using 4 subintervals of equal width. Which of the following lists the estimations from least to greatest?

(A) $R < T < L$ (B) $R < L < T$ (C) $T < L < R$ (D) $L < R < T$ (E) $L < T < R$

$y = 30 - x^3$
$= -x^3 + 30$



$R < T < L$

B 5. The function $h(x)$ is continuous on the interval $[-4, 12]$. Selected values of x and $f(x)$ are given in the table below. If $\int_{-4}^{12} f(x)\,dx$ is estimated using a right Riemann sum with 4 equal subintervals, a left Riemann sum with 4 equal subintervals, trapezoids with 4 equal subintervals, and a midpoint Riemann sum with 2 equal subintervals, what is the difference between the largest and smallest estimation?

4 4 4 4

x	-4	0	4	8	12
$f(x)$	3	9	-2	-6	-3

(A) 68 (B) 32 (C) 24 (D) 16 (E) 8

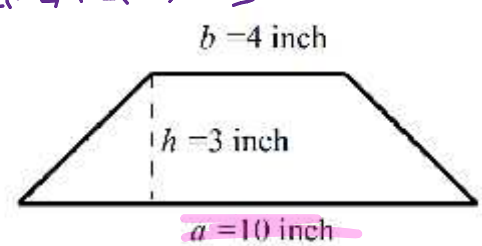
$R_4 = 4[9 - 2 - 6 - 3] = -8$

$L_4 = 4[3 + 9 - 2 - 6] = 16$

$T_4 = \frac{1}{2}(4)[3 + 2(9) + 2(-2) + 2(-6) - 3] = 4$

$M_2 = 8[9 - 6] = 24$

$24 - (-8) = \boxed{32}$



C 6. A trapezoid is pictured above. It has a base, a, that is a constant 10 inches while its top base, b, is increasing at a rate of 3 inches per minute while its height, h, is decreasing at a rate of $\frac{1}{2}$ inches per minute. When the top base is 4 inches and the height is 3 inches, how fast, in square inches per minute, is the area of the trapezoid changing?

(A) 8 (B) 2 (C) 1 (D) $-\frac{3}{4}$ (E) $-\frac{5}{2}$

$\frac{db}{dt} = 3$

$\frac{dh}{dt} = -\frac{1}{2}$

$\boxed{b=4,\ h=3,\ a=10}$

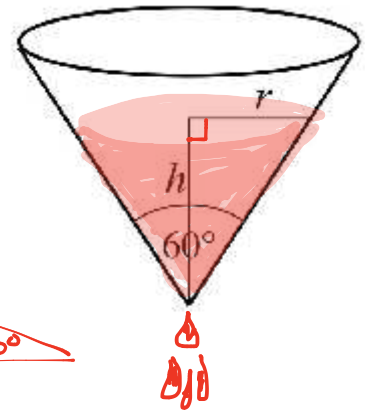
$\frac{dA}{dt} = ?$

$A = \frac{1}{2}(10 + b)h$

$\frac{dA}{dt} = \frac{1}{2}\left[\frac{db}{dt}h + (10 + b)\frac{dh}{dt}\right]$

when $b=4$: $\frac{dA}{dt} = \frac{1}{2}\left[(3)(3) + (10+4)(-\frac{1}{2})\right] = \frac{1}{2}[9 - 7] = 1\ \text{in}^2/\text{min}$

D 7. Kool-Aid is draining from a conical tank whose base angle is $60°$ as shown in the figure at the right. When the height of the Kool-Aid is 3 feet, its height is decreasing at 6 inches per hour. At this moment, how fast, in cubic feet per hour, is the volume of the Kool-Aid decreasing?



(A) 162π (B) 18π (C) $\frac{13\pi}{2}$ (D) $\frac{3\pi}{2}$ (E) 3π

$\boxed{h = 3\text{ ft}}$ $\boxed{r = \frac{3}{\sqrt{3}}}$

$\frac{dh}{dt} = -6 \text{ in/hr}$

$\frac{dh}{dt} = -\frac{1}{2} \text{ ft/hr}$

$\frac{dV}{dt} = ?$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{\sqrt{3}} h\right)^2 h$

$\boxed{V = \frac{\pi}{9} h^3}$

$\boxed{\frac{dV}{dt} = \frac{\pi}{3} h^2 \frac{dh}{dt}}$

when $h = 3$ ft

$\frac{dV}{dt} = \frac{\pi}{3}(3^2)\left(-\frac{1}{2}\right)$

$\frac{dV}{dt} = 3\pi\left(-\frac{1}{2}\right)$

$= -\frac{3\pi}{2} \text{ ft}^3/\text{hr}$

So the volume is decreasing by $\boxed{\frac{3\pi}{2} \text{ ft}^3/\text{hr}}$

2, 1, 30, $\sqrt{3}$

r, 30°, h

$\frac{r}{h} = \frac{1}{\sqrt{3}}$

$\boxed{r = \frac{1}{\sqrt{3}} h \text{ or } h = \sqrt{3} r}$

A 8. $\int \frac{\pi}{x^e} dx =$

(A) $\frac{\pi x^{1-e}}{1-e} + C$ (B) $\frac{\pi}{(e+1)x^{e+1}} + C$ (C) $\frac{\pi}{x^{e+1}} + C$ (D) $\pi x^{1-e} + C$ (D) $\frac{\pi x^{e+1}}{e+1} + C$

$\pi \int x^{-e} dx$

$\pi\left[\frac{x^{-e+1}}{-e+1}\right] = \frac{\pi \cdot x^{1-e}}{1-e}$

B 9. Use a tangent line approximation for $g(x) = \sqrt{x}$ at $x = 64$ to estimate $\sqrt{65} - \sqrt{63}$.

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{16}$ (D) $\frac{1}{32}$ (E) 0

$\cancel{x}$ $\sqrt{65} - \sqrt{63}$

$= 0.1250038151$

& $\frac{1}{8} = 0.125$

(Darn close!)

$g(64) = 8$, pt: $(64, 8)$

$g' = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$g'(64) = \frac{1}{16} = m$

$L(x) = 8 + \frac{1}{16}(x - 64)$

$\sqrt{65} \approx L(65) = 8 + \frac{1}{16}$

$\sqrt{63} \approx L(63) = 8 - \frac{1}{16}$

$\sqrt{65} - \sqrt{63} \approx \left[8 + \frac{1}{16}\right] - \left[8 - \frac{1}{16}\right]$

$= \frac{2}{16}$

$= \frac{1}{8}$

E 10. $\int \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} dx =$

(A) $\frac{(\sqrt{x}-1)^3}{3\sqrt{x}} + C$ (B) $\frac{(\sqrt{x}-1)^3}{3} + C$ (C) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$

(D) $\frac{1}{2}x^{1/2} - \frac{4}{3}x + x^{1/2} + C$ (E) $\frac{2}{3}x^{3/2} - 2x + 2x^{1/2} + C$

$\int \left(\frac{x - 2\sqrt{x} + 1}{\sqrt{x}}\right) dx$

$\int \left(\frac{x}{x^{1/2}} - \frac{2\sqrt{x}}{\sqrt{x}} + \frac{1}{x^{1/2}}\right) dx$

$\int \left(x^{1/2} - 2 + x^{-1/2}\right) dx$

$\frac{2}{3}x^{3/2} - 2x + 2x^{1/2} + C$

1 C 2 D 3 D 4 A 5 B 6 C 7 D 8 A 9 B 10 E

PART II: Free Response—Use Proper Notation

11. I was out collecting data yesterday and used it to approximate a **differentiable** function $y = f(x)$ represented in the table below.

4 4 3 3 1 1

x	0	4	8	11	14	15	16
y	30	6	1	2	0	-1	0

. . . use my data to **approximate** $I = \int_0^{16} f(x)dx$ using the following methods using the given number of subintervals, n. (**simplify your answers**):

(a) Left end-point Riemann Sums ($n = 6$).

$I \approx L_6 = 4(30)+4(6)+3(1)+3(2)+1(0)+1(-1)$ √1

$= 152$ √2

(b) Right end-point Riemann Sums ($n = 6$)

$I \approx R_6 = 4(6)+4(1)+3(2)+3(0)+1(-1)+1(0)$ √3

$= 33$ √4

(c) Midpoint Riemann Sums ($n = 3$)

$I \approx M_3 = 8(6)+6(2)+2(-1)$ √5

$= 58$ √6

(d) Trapezoidal Rule ($n = 6$)

$I \approx \frac{L_6 + R_6}{2} = \frac{152+33}{2} = 92.5$ √7

or

$I \approx \frac{1}{2}\left[4(30+6)+4(6+1)+3(1+2)+3(2+0)+1(0+-1)+1(-1+0)\right] = 92.5$

(e) Can any of the above calculations represent the approximate area under the function $y = f(x)$ on $[0,16]$? Why or why not?

No, since $f(15)<0$ √8 and/or there may be more negative y-values on the interval that are not listed in the table.

(f) **Approximate** $f'(12)$ from the table of values. Make sure to show your difference quotient.

$f'(12) \approx \frac{0-2}{14-11} = \frac{-2}{3}$ √9

or $\frac{2-0}{11-14}$

(g) If the **secant** line on the interval $[11,14]$ was used to approximate $f(12)$, given that $f'(x) < 0$ and $f''(x) < 0$ for all $x \in (11,14)$, would this approximation of $f(12)$ be an over or under approximation? Explain why..

$= S(x)$ dec concave down

This would be an UNDER approximation since the secant line is BELOW the curve of $f(x)$ on $(11,14)$. √10