Name KEY Date__________ Campaign Promise________

TEST: AP Calculus: Test—3.6-4.2. No Calculator

PART I: Multiple Choice. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

D 1. $\int \frac{x+1}{x^2}dx = \int\left(\frac{x}{x^2}+\frac{1}{x^2}\right)dx = \int\left(\frac{1}{x}+x^{-2}\right)dx = \ln|x| - 1x^{-1} + C = -x^{-1} + \ln|x| + C$

(A) $\ln x^2 + \ln|x| + C$ (B) $-\ln x^2 + \ln|x| + C$ (C) $x^{-1} + \ln|x| + C$

(D) $-x^{-1} + \ln|x| + C$ (E) $-2x^3 + \ln|x| + C$

B 2. $\int\left(x^3+1\right)^2 dx = \int\left(x^6+2x^3+1\right)dx = \frac{1}{7}x^7 + \frac{2}{4}x^4 + x + C = \frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(A) $\frac{1}{7}x^7 + x + C$ (B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$ (C) $6x^2\left(x^3+1\right) + C$

(D) $\frac{1}{3}\left(x^3+1\right)^3 + C$ (E) $\frac{\left(x^3+1\right)^3}{9x^2} + C$

B 3. The most general antiderivative of $f(x) = \frac{\cos x}{1-\cos^2 x}$ is

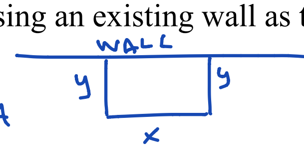
(A) $\csc x + C$ (B) $-\csc x + C$ (C) $\cot x + C$ (D) $-\cot x + C$ (E) $\csc x \cot x + C$

$\int f(x)dx = \int \frac{\cos x}{1-\cos^2 x}dx$

$F(x) = \int \frac{\cos x}{\sin^2 x}dx = \int \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}dx = \int \cot x\cdot\csc x\,dx = \int \csc x\cdot\cot x\,dx$

$= -\csc x + C$

D 4. Given 4 feet of fencing, what is the maximum number of square feet that can be enclosed if the fencing is used to make three sides of a rectangular pen, using an existing wall as the fourth side? (A) $\frac{32}{9}$ (B) 1 (C) $\frac{16}{9}$ (D) 2 (E) 4



Constraint
$2y + x = 4$
$x = 4 - 2y$

Maximize Area, A
$A = xy$
$A = (4-2y)y$
$A = 4y - 2y^2$ ← parabola opening down
$A' = 4 - 4y = 0$ → Abs max @ $y=1$
$4 = 4y$, $\boxed{y=1}$

So $A(1) = (4-2)(1) = 2$

A 5. If $\int_0^5 f(x)dx = -3$ and $\int_{-2}^5 f(x)dx = 6$, find $\int_{-2}^0 3f(x)dx$

(A) 27 (B) 9 (C) -27 (D) -9 (E) 18

$3\int_{-2}^0 f(x)dx = 3\left[\int_{-2}^5 f(x)dx + \int_5^0 f(x)dx\right]$

$= 3\left[6+3\right]$

$= 3\cdot 9$

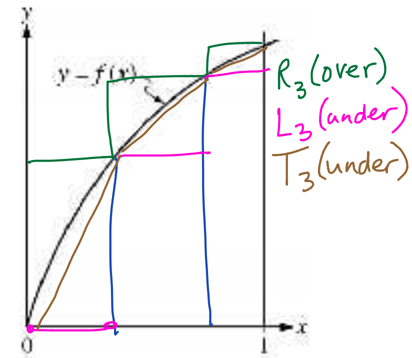
$= 27$

E 6. The approximate value of $y = x^2$ at $x = 2$, obtained from the tangent to the graph at $x = 1$ is

(A) 3.1 (B) 5 (C) 4.2 (D) 4 (E) 3

$y(1) = 1^2 = 1$, pt: $(1,1)$
$y'(x) = 2x$, $y'(1) = 2 = m$
so, $L(x) = 1 + 2(x-1)$
$y(2) \approx L(2) = 1 + 2(2-1) = 1 + 2 = 3$

D 7. A left Rieman sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)dx$, each using the same number of subintervals. The graph of the function f is shown at right. Which of the sums give an underestimate of the value of $\int_0^1 f(x)dx$?

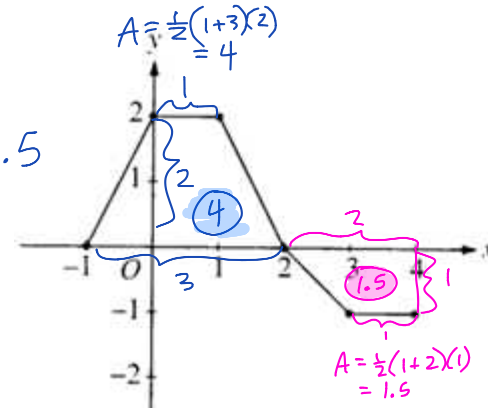


I. Left Sum II. Right Sum III. Trapezoidal sum

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

B 8. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown **at right**. What is the value of $\int_{-1}^{4} f(x)dx$? $= 4 - 1.5 = 2.5$

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8



C 9. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to THREE times the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $3\sqrt{2}$ (B) 1.5 (C) 3 (D) 6 (E) 3π

$\frac{dA}{dt} = 3\frac{dC}{dt}$

$A = \pi r^2$ $\quad$ $C = 2\pi r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad$ $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$ $\quad$ so, $\cancel{2\pi} r \cancel{\frac{dr}{dt}} = 3(\cancel{2\pi} \cancel{\frac{dr}{dt}})$

$\boxed{r = 3}$

$\quad\quad\quad\quad 2 \quad\quad 2 \quad\quad 2$

x	0	2	4	6
$f(x)$	4	k	8	12

D 10. The function f is continuous on the closed interval $[0,6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

$\int_0^6 f(x)dx \approx \left(\frac{1}{2}\right)(2)\left[4 + 2k + 2(8) + 12\right] = 52$
$4 + 2k + 16 + 12 = 52$
$2k + 32 = 52$
$2k = 20$
$k = 10$

PART II: Free Response.
Show all work in the space provided below the line.

11. I was out collecting data yesterday and tried to use it to approximate a **differentiable** function $y = f(x)$ represented in the table below.

2 1 3 2 1 don't use

x	0	2	3	6	8	9	~~10~~
y	1	0	2	3	-1	4	~~5~~

. . . for parts (a) through (c) use my data to **approximate** $\int_0^9 f(x)dx$ using 5 subintervals as indicated by the data using the following methods. (**use correct notation, simplify your answers, and indicate your method**):

(a) Left end-point Riemann Sums ($n = 5$).

$$\int_0^9 f(x)\,dx \approx L_5 = 2(1)+1(0)+3(2)+2(3)+1(-1) \quad \checkmark 1$$
$$= 2 + 0 + 6 + 6 - 1$$
$$= 13 \quad \checkmark 2$$

(b) Right end-point Riemann Sums ($n = 5$)

$$\int_0^9 f(x)\,dx \approx R_5 = 2(0)+1(2)+3(3)+2(-1)+1(4) \quad \checkmark 3$$
$$= 0 + 2 + 9 - 2 + 4$$
$$= 13 \quad \checkmark 4$$

(c) Trapezoidal Rule ($n = 5$)

$$\int_0^9 f(x)\,dx \approx \frac{1}{2}\left[2(1+0)+1(0+2)+3(2+3)+2(3-1)+1(-1+4)\right] \quad \checkmark 5$$
$$= \frac{1}{2}\left[2+2+15+4+3\right]$$
$$= \frac{1}{2}(26)$$
$$= 13 \quad \checkmark 6$$

$$\boxed{\text{or } \frac{L_5+R_5}{2} = \frac{13+13}{2} = 13}$$

(d) Can any of the above calculations represent the approximate area under the function $y = f(x)$ on $[0, 9]$? Why or why not? Be specific.

No, since $y(8) = -1 < 0$

√7

(e) **Approximate** $f'(5)$ from the table of values. Simplify your answer. Show the work that leads to your answer.

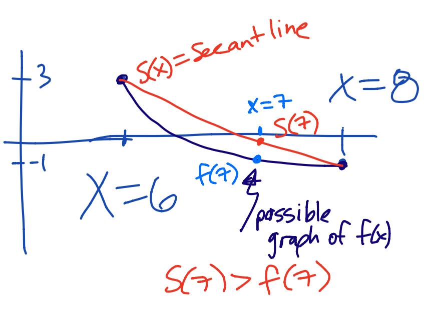
$f'(5) \approx \frac{3-2}{6-3} = \frac{1}{3}$ or $\frac{2-3}{3-6} = \frac{-1}{-3} = \frac{1}{3}$

√8

(f) If the **secant** line on the interval $[6,8]$ was used to approximate $f(7)$, given that $f'(x) < 0$ and $f''(x) > 0$ for all $x \in (6,8)$, would this approximation of $f(7)$ be an over or under approximation? Explain why.

Decreasing

concave up



The approximation would be an OVER approximation since f(x) is concave UP on the interval $t \in (6,8)$

√9

√10