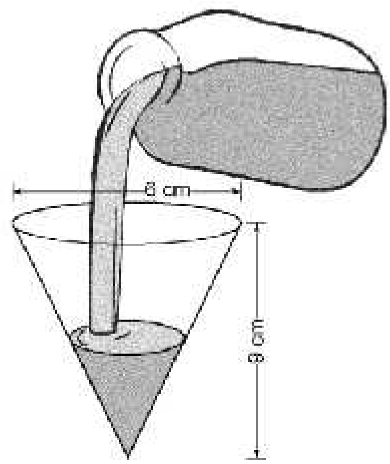
Name________________________________________________ Date________________ Cattle Brand___________

AP Calculus TEST 3.6-4.1, No Calculator

Section I: Multiple Choice—put the CAPITAL letter of the correct answer choice to the left of each question number.

_____ 1. A conical-shaped paper cup is shown in the diagram below.




If water cranberry juice is poured into the cup at a rate of 1 cubic centimeter per second, how fast is the depth of the cranberry juice in the cup increasing when the juice is 4 cm deep?

(A) $\frac{16\text{®}}{9}$ cm/sec (B) $\frac{9}{64\text{®}}$ cm/sec (C) $\frac{9}{16\text{®}}$ cm/sec (D) $\frac{64\text{®}}{9}$ cm/sec (E) $\frac{16}{9\text{®}}$ cm/sec

_____ 2. Let $f$ be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line at $x=3$ is used to find an approximation to a zero of $f$, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

______ 3. $\int \frac{3x^5+2x^3-x^2}{x^2}\,dx =$

(A) $18x^6+8x^2-2x+C$ (B) $\frac{3}{4}x^4+x^2-x+C$ (C) $\frac{15x^4+6x^2-2x}{2x}+C$

(D) $\frac{x^6+x^4-x^3}{6x^3}+C$ (E) $3x^4+2x^2-x+C$

______ 4. At each point $(x, y)$ on a curve, $\frac{d^2y}{dx^2}=6x$. Additionally, the line $y=6x+4$ is tangent to the curve at $x=-2$. Which of the following Is an equation fo the curve that satisfies these conditions?

(A) $y=6x^2-32$ (B) $y=2x^3+3x-12$ (C) $y=2x^3-3x$ (D) $y=x^3-6x-12$ (E) $y=x^3-6x+12$

______ 5. $\int \frac{\sin 2x}{\cos x}\,dx =$

(A) $\cos x+C$ (B) $-2\cos x+C$ (C) $-\cos 2x+C$ (D) $2\cos x+C$ (E) $\cos 2x+C$

______ 6. The sum of two positive integers is 90. If the product of one integer and the square of the other is a maximum, the the larger integer is

(A) 75 (B) 50 (C) 30 (D) 60 (E) 80

_____ 7. $\int\left(x^2-2\right)^2 dx =$

(A) $\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$ (B) $\frac{\left(x^2-2\right)^3}{6x}+C$ (C) $\left(\frac{x^3}{3}-2x\right)^2+C$

(D) $\frac{2x}{3}\left(x^2-2\right)^3+C$ (E) $\frac{x^5}{5}+4x+C$

_____ 8. Which of the following defines a function $f$ such that $f'(x)=\sqrt{x}$ with the initial condition $f(9)=0$?

(A) $f(x)=\frac{2}{3}x\sqrt{x}-18$ (B) $f(x)=\frac{x\sqrt{x}}{3}+9$ (C) $f(x)=x\sqrt{x}-3x$

(D) $f(x)=\frac{1}{2}\sqrt{x}-3$ (E) $f(x)=\frac{3}{2}x\sqrt{x}-18$

_____ 9. The radius of a spherical ball Is decreasing at a constant rate of 3 centimeters per second. Find, In cubic centimeters per second, the rate of change of the volume of the ball when the radius is 5 cm.

(A) −60® (B) −150® (C) −300® (D) −100® (E) −12®

Part II: Free Response—Show all work in the space provided

10. Let $\frac{d^2y}{dx^2} = -3x^2 - 4$ for some particular function $y = f(x)$.

(a) If $y'(1) = 5$ and $y(1) = -\frac{1}{4}$, find the particular solution $y = f(x)$. Show the work that leads to your answer with correct notation.

(b) Write an equation for the tangent line to the particular solution $y = f(x)$ at $x = 1$.

(c) Use your equation from part (b) to approximate $f(1.2)$. Simplify your answer.

(d) Is your approximation from part (c) and over- or an under-approximation? Justify.