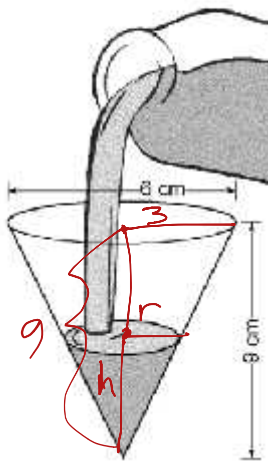
Name KEY ______________________ Date ____________ Cattle Brand __________

AP Calculus TEST 3.6-4.1, No Calculator

Section I: Multiple Choice—put the CAPITAL letter of the correct answer choice to the left of each question number.

C 1. A conical-shaped paper cup is shown in the diagram below.



$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{3}h\right)^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{9}h^2\right)h$

$V = \frac{\pi}{27} h^3$

$\frac{r}{h} = \frac{3}{9}$

$\boxed{r = \frac{1}{3}h \quad h = 3r}$

$\frac{d}{dt}: \frac{dV}{dt} = \frac{\pi}{9}h^2 \cdot \frac{dh}{dt}$

When $h=4$: $1 = \frac{\pi}{9}(4^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{16\pi}$ cm/sec

If water cranberry juice is poured into the cup at a rate of 1 cubic centimeter per second, how fast is the depth of the cranberry juice in the cup increasing when the juice is 4 cm deep?

(A) $\frac{16\pi}{9}$ cm/sec (B) $\frac{9}{64\pi}$ cm/sec (C) $\frac{9}{16\pi}$ cm/sec (D) $\frac{64\pi}{9}$ cm/sec (E) $\frac{16}{9\pi}$ cm/sec

$\frac{dV}{dt} = 1$, $\frac{dh}{dt} = ?$, $\boxed{h=4}$

C 2. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

eq: $y = 2 + 5(x-3) = 0$

$x - 3 = -\frac{2}{5}$

$x = 3 - \frac{2}{5}$

$x = \frac{13}{5} = 2.6$

B 3. $\int \frac{3x^5+2x^3-x^2}{x^2}dx =$

(A) $18x^6+8x^2-2x+C$ (B) $\frac{3}{4}x^4+x^2-x+C$ (C) $\frac{15x^4+6x^2-2x}{2x}+C$

(D) $\frac{x^6+x^4-x^3}{6x^3}+C$ (E) $3x^4+2x^2-x+C$

$\int\left[\frac{3x^5}{x^2}+\frac{2x^3}{x^2}-\frac{x^2}{x^2}\right]dx$

$\int\left[3x^3+2x-1\right]dx$

$\frac{3}{4}x^4+x^2-x+C$

D 4. At each point (x,y) on a curve, $\frac{d^2y}{dx^2}=6x$. Additionally, the line $y=6x+4$ is tangent to the curve at $x=-2$. Which of the following Is an equation fo the curve that satisfies these conditions?

(A) $y=6x^2-32$ (B) $y=2x^3+3x-12$ (C) $y=2x^3-3x$ (D) $y=x^3-6x-12$ (E) $y=x^3-6x+12$

at $x=-2$: $y'(-2)=6$ (slope of tangent line)

$y(-2)=6(-2)+4$
$=-12+4$
$=-8$ (y-value of tangent line)

$\frac{dy}{dx}=3x^2+C$

for $y'(-2)=6$: $6=3(-2)^2+C$
$6=12+C$
$C=-6$

So $\frac{dy}{dx}=3x^2-6$

$y=x^3-6x+C$

for $y(-2)=-8$: $-8=(-2)^3-6(-2)+C$
$-8=-8+12+C$
$C=-12$

So, $y=x^3-6x-12$

B 5. $\int \frac{\sin 2x}{\cos x}dx =$

(A) $\cos x+C$ (B) $-2\cos x+C$ (C) $-\cos 2x+C$ (D) $2\cos x+C$ (E) $\cos 2x+C$

$\int \frac{2\sin x\cos x}{\cos x}dx$

$\int 2\sin x\,dx$

$-2\cos x+C$

D 6. The sum of two positive integers is 90. If the product of one integer and the square of the other is a maximum, the the larger integer is

(A) 75 (B) 50 (C) 30 (D) 60 (E) 80

$a+b=90$, $a,b\in\mathbb{Z}^+$

$\boxed{a=9-b} \rightarrow ab^2=P$

$(90-b)b^2=P$

$P=90b^2-b^3$, $b\in(0,90)$

$P'=180b-3b^2=0$

$3b(60-b)=0$

$\cancel{b=0}$, $b=60$

So, $b=60$
&
$a=90-60=30$

So, larger number is $b=60$

A 7. $\int (x^2-2)^2\,dx =$

(A) $\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$ (B) $\frac{(x^2-2)^3}{6x}+C$ (C) $\left(\frac{x^3}{3}-2x\right)^2+C$

(D) $\frac{2x}{3}(x^2-2)^3+C$ (E) $\frac{x^5}{5}+4x+C$

$\int [x^4-4x^2+4]\,dx$

$\frac{1}{5}x^5-\frac{4}{3}x^3+4x$

$\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$

A 8. Which of the following defines a function f such that $f'(x)=\sqrt{x}$ with the initial condition $f(9)=0$?

(A) $f(x)=\frac{2}{3}x\sqrt{x}-18$ (B) $f(x)=\frac{x\sqrt{x}}{3}+9$ (C) $f(x)=x\sqrt{x}-3x$

(D) $f(x)=\frac{1}{2}\sqrt{x}-3$ (E) $f(x)=\frac{3}{2}x\sqrt{x}-18$

$f'=x^{1/2}$

$f=\frac{2}{3}x^{3/2}+C$

$f=\frac{2}{3}(\sqrt{x})^3+C$

for $f(9)=0$: $0=\frac{2}{3}(\sqrt{9})^3+C$

$0=\frac{2}{3}(27)+C$

$0=18+C$

$C=-18$

So,

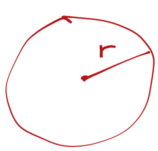
$f(x)=\frac{2}{3}x^{3/2}-18$

$f(x)=\frac{2}{3}x\cdot x^{1/2}-18$

$f(x)=\frac{2}{3}x\sqrt{x}-18$

C 9. The radius of a spherical ball Is decreasing at a constant rate of 3 centimeters per second. Find, In cubic centimeters per second, the rate of change of the volume of the ball when the radius is 5 cm.

(A) -60π (B) -150π (C) -300π (D) -100π (E) -12π



$\frac{dr}{dt}=-3$

$\frac{dV}{dt}=?$

$\boxed{r=5}$

$V=\frac{4}{3}\pi r^3$

$\frac{d}{dt}$: $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

when $r=5$: $\frac{dV}{dt}=4\pi(5^2)(-3)$

$=-300\pi\ \text{cm}^3/\text{sec}$

Part II: Free Response—Show all work in the space provided

10. Let $\frac{d^2y}{dx^2} = -3x^2 - 4$ for some particular function $y = f(x)$.

(a) If $y'(1) = 5$ and $y(1) = -\frac{1}{4}$, find the particular solution $y = f(x)$. Show the work that leads to your answer with correct notation.

$$\frac{d^2y}{dx^2} = -3x^2 - 4$$

$$\frac{dy}{dx} = -x^3 - 4x + C \quad \sqrt{1}$$

for $y'(1)=5$: $5 = -1 - 4 + C$

$C = 10 \quad \sqrt{2}$

So, $\frac{dy}{dx} = -x^3 - 4x + 10$

$$\frac{dy}{dx} = -x^3 - 4x + 10$$

$$y = -\frac{1}{4}x^4 - 2x^2 + 10x + D \quad \sqrt{3}$$

for $y(1) = -\frac{1}{4}$: $-\frac{1}{4} = -\frac{1}{4} - 2 + 10 + D$

$0 = 8 + D$

$D = -8 \quad \sqrt{4}$

So, $y = -\frac{1}{4}x^4 - 2x^2 + 10x - 8 \quad \sqrt{5}$

(b) Write an equation for the tangent line to the particular solution $y = f(x)$ at $x = 1$.

pt: $(1, -\frac{1}{4})$

m: 5

eq: $y = -\frac{1}{4} + 5(x-1) \quad \sqrt{6}$

(c) Use your equation from part (b) to approximate $f(1.2)$. Simplify your answer.

$$f(1.2) \approx y(1.2) = -\frac{1}{4} + 5(1.2 - 1)$$

$$= -\frac{1}{4} + 5(0.2)$$

$$= -\frac{1}{4} + 1$$

$$= \frac{3}{4} \text{ or } 0.75 \quad \sqrt{7}$$

¡SQUIGGLE ALERT!

(d) Is your approximation from part (c) and over- or an under-approximation? Justify.

from part (a): $\frac{d^2y}{dx^2} = -3x^2 - 4 < 0 \;\; \forall x \in \mathbb{R}$

So, $y = f(x)$ is concave down $\forall x \in \mathbb{R}$ $\quad \sqrt{8}$

So, tangent lines are above the curve $y = f(x)$

So, $y(1.2)$ OVERAPPROXIMATES $f(1.2)$

$\sqrt{9}$