Name KEY Date_________ Winter Ailment_____________

AP Calculus TEST: 3.1 to 4.1, **No Calculator**

I. Multiple Choice: Put the correct CAPITAL letter in the blank to the left of the question number.

C 1. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line at $x=3$ is used to find an approximation to a zero of f. That approximation is which of the following?

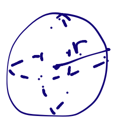
(A) 0.4
(B) 0.5
(C) 2.6
(D) 3.4
(E) 5.5

pt: $(3,2)$
Slope $=5$
So, $\mathcal{L}(x)=2+5(x-3)$

$\mathcal{L}(x)=0$
$2+5(x-3)=0$
$5x-15+2=0$
$5x=13$
$x=\frac{13}{5}=2.6$

C 2. The radius of a spherical ball is decreasing at a constant rate of 3 cm per second. Find, in cubic centimeters per second, the rate of change of the volume of the ball when the radius is 5 cm.

(A) -60π
(B) -150π
(C) -300π
(D) -12π
(E) -100π



$\frac{dr}{dt}=-3$
$\frac{dV}{dt}=?$
$\boxed{r=5}$

$V=\frac{4}{3}\pi r^3$
$\frac{d}{dt}$: $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$
when $r=5$: $\frac{dV}{dt}=4\pi(5)^2(-3)$
$=-300\pi$ cm³/sec

B 3. If $f'(x)=\sqrt[4]{x^3}-\frac{2}{\sqrt[3]{x^2}}$, then what is $f(x)$, the general antiderivative of $f'(x)$?

(A) $f(x)=\frac{7}{4}x\cdot\sqrt[4]{x^3}-\frac{5}{6}x\cdot\sqrt[3]{x^2}+C$

(B) $f(x)=\frac{4}{7}x\cdot\sqrt[4]{x^3}-6\cdot\sqrt[3]{x}+C$

(C) $f(x)=\frac{4}{7}x\cdot\sqrt[4]{x^3}-\frac{6}{5}x\cdot\sqrt[3]{x^2}+C$

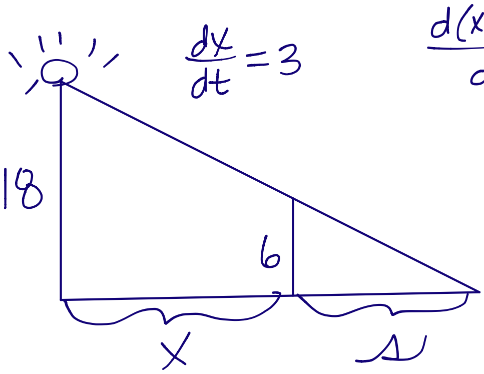
(D) $f(x)=\frac{4}{7}\sqrt[4]{x^3}-\frac{2}{3}\sqrt[3]{x}+C$

(E) $f(x)=\frac{3}{4\sqrt[4]{x}}-\frac{4}{3\sqrt[3]{x}}+C$

$f'(x)=x^{3/4}-2x^{-2/3}$
$f(x)=\frac{4}{7}x^{7/4}-(2)(3)x^{1/3}+C$
$=\frac{4}{7}\sqrt[4]{x^7}-6\sqrt[3]{x}+C$
$=\frac{4}{7}\sqrt[4]{x^4\cdot x^3}-6\sqrt[3]{x}+C$
$=\frac{4}{7}x\cdot\sqrt[4]{x^3}-6\sqrt[3]{x}+C$

C 4. A street light is hung 18 feet above street level. A 6-foot tall man standing directly under the light walks away at a rate of 3 ft/sec. How fast is the tip of the man's shadow moving?

(A) $\frac{7}{2}$ ft/sec
(B) 3 ft/sec
(C) $\frac{9}{2}$ ft/sec
(D) $\frac{1}{2}$ ft/sec
(E) $\frac{3}{2}$ ft/sec



$\frac{dx}{dt}=3$

$\frac{d(x+s)}{dt}=\frac{dx}{dt}+\frac{ds}{dt}$

$\frac{18}{6}=\frac{x+s}{s}$
$3s=x+s$
$2s=x$

So, $2s=x$
$s=\frac{1}{2}x$
$\frac{d}{dt}$: $\frac{ds}{dt}=\frac{1}{2}\frac{dx}{dt}$
So, $\frac{ds}{dt}=\frac{1}{2}(3)$
$=\frac{3}{2}$
So $\frac{d(x+s)}{dt}=\frac{dx}{dt}+\frac{ds}{dt}$
$=3+\frac{3}{2}$
$=\frac{9}{2}$ ft/sec

D 5. The graph of $f(x) = 8x^5 - 5x^4$ will have how many points of inflection?

(A) Four
(B) Two
(C) Three
(D) One
(E) None

$f'(x) = 40x^4 - 20x^3$
$f''(x) = 160x^3 - 60x^2$
$f'' = \text{DNE}$ NEVER; $f'' = 0$
$160x^3 - 60x^2 = 0$
$20x^2(8x-3) = 0$
$x = 0,\ x = \frac{3}{8}$ ← P.I.V.s

x	-1	0	$3/16$	$3/8$	1
f''	$-$		$-$		$+$

So, f has only one inflection point at $x = \frac{3}{8}$

A 6. The sum of two positive integers is 90. If the product of one integer and the square of the other is a maximum, then the greater integer is

(A) 60
(B) 50
(C) 75
(D) 55
(E) 80

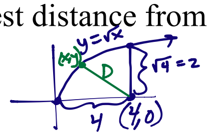
constraint
$x + y = 90$
$\boxed{x = 90 - y}$

$P = xy^2$
$P = (90-y)y^2$
$P = 90y^2 - y^3,\ y > 0$
$P' = 180y - 3y^2 = 0$
$3y(60-y) = 0$

So $y = 0$ (not pos) or $\boxed{y = 60}$
& $x = 90 - y$
$x = 90 - 60$
$\boxed{x = 30}$
So, larger integer is 60

A 7. The shortest distance from the curve $y = \sqrt{x}$ to the point $(4,0)$ is

(A) $\frac{\sqrt{15}}{2}$
(B) $\frac{\sqrt{14}}{2}$
(C) $\frac{7}{2}$
(D) $\sqrt{15}$
(E) $\sqrt{14}$



constraint

$D = \sqrt{(x-4)^2 + (y-0)^2}$
$D = \sqrt{(x-4)^2 + y^2}$
So, $D = \sqrt{(x-4)^2 + (\sqrt{x})^2},\ x \in [0,4]$
$D = \sqrt{x^2 - 8x + 16 + x}$
$D = \sqrt{x^2 - 7x + 16}$
$D' = \frac{2x-7}{2\sqrt{x^2-7x+16}} = 0$
$2x - 7 = 0$
$x = \frac{7}{2}$

$D(0) = 4$
$D\left(\frac{7}{2}\right) = \sqrt{\left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) + 16} = \sqrt{\frac{49}{4} - \frac{49}{2} + \frac{64}{4}} = \sqrt{-\frac{49}{4} + \frac{64}{4}} = \sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{2}$
$D(4) = 2$

$\sqrt{\frac{1}{4} + \frac{7}{2}} = \sqrt{\frac{15}{2}}$

D 8. Which of the given functions does NOT satisfy the conditions of the Mean Value Theorem on the interval $x \in [-2, 2]$?

I. $f(x) = \frac{x^4}{4} - \frac{x^2}{2} - x$ Polynomial ✓

II. $f(x) = x^{2/3} - \frac{3}{x^2}$

III. $f(x) = x^2 + \frac{1}{2x}$

not continuous or diffable at $x = 0 \in [-2,2]$
so, MVT does not Apply

(A) I, II, and III (B) I and III only (C) II only (D) II and III only (E) III only

B 9. The function defined by $f(x) = 8x^2 - 2x^4$ has

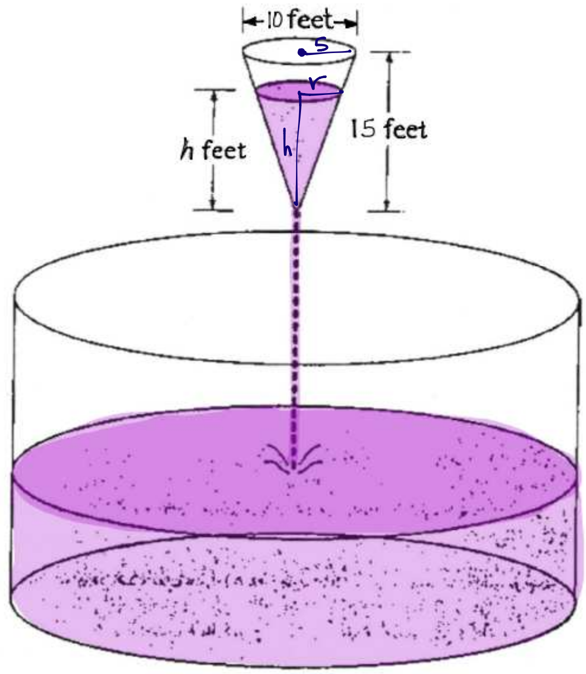
(A) No local extrema
(B) Two local maxima and one local minimum
(C) Two local maxima and two local minima
(D) Two local minima and one local maximum
(E) One local maximum and one local minimum

$f'(x) = 16x - 8x^3$
$f' = \text{DNE}$ NEVER; $f' = 0$
$16x - 8x^3 = 0$
$8x(2 - x^2) = 0$
$x = 0,\ x = \pm\sqrt{2}$

x	-2	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$	2
f'	$+$		$-$		$+$		$-$
		∩		∪		∩	

f has 2 local maxs & 1 local min

II. Free Response: Show all work in the space provided using correct notation. Include units on all final answers.



10. As shown in the figure above, Kool-Aid is draining from a conical tank with height 15 feet and diameter 10 feet into a cylindrical tank that has a base with area 900π square feet. The depth, h, in feet, of the Kool-Aid in the conical tank is changing at the rate of $\frac{dh}{dt} = h - 12$ feet per minute.

(a) Write an expression for the volume of Kool-Aid in the conical tank as a function of h.

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3} \left(\frac{1}{3}h\right)^2 h$

$V = \frac{\pi}{27} h^3$ ✓2

$\frac{r}{h} = \frac{5}{15}$

$\boxed{\begin{array}{l} r = \frac{1}{3}h \\ h = 3r \end{array}}$ ✓1

(b) At what rate is the volume of Kool-Aid in the conical tank changing when $h = 3$? Indicate units of measure. **Write a sentence, with units, explaining what your answer means in the context of the problem.**

$\frac{dV}{dt} = ?$

$\boxed{h=3}$

$V = \frac{\pi}{27} h^3$

$\frac{d}{dt}: \frac{dV}{dt} = \frac{3\pi}{27} h^2 \frac{dh}{dt}$ √3

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$

when $h=3$: $\frac{dV}{dt} = \frac{\pi}{9}(3)^2(3-12)$ √4

$= \pi(-9)$

$= -9\pi \ ft^3/min$

When the height is 3 feet, the Volume is decreasing by 9π cubic feet per min. √5

(c) Let y be the depth, in feet, of the Kool-Aid in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

$V = \pi R^2 H$

So

$V = \pi (30)^2 y$ (constant)

$V = 900\pi y$ √6

Base is a cylinder So Area of base:

$A = 900\pi = \pi r^2$

$r^2 = 900$

$r = 30$ feet

$* \ \frac{dV}{dt} = +9\pi$

$\frac{d}{dt}: \frac{dV}{dt} = 900\pi \frac{dy}{dt}$ √7

when $h=3$: $9\pi = 900\pi \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9\pi}{900\pi}$

$= \frac{1}{100} \ ft/min.$

√8

units on b & c √9