Name KEY Date________ Best Xmas Gift______________

AP Calculus TEST: 3.1 to 3.8, **No Calculator**

I. Multiple Choice: Put the correct CAPITAL letter in the blank to the left of the question number.

A 1. Find the value for which $f(x)=x^2+7$ on the interval $1\le x\le 3$ satisfies the Mean Value Theorem.

(A) 2

(B) $\frac{9}{4}$

(C) $\frac{7}{3}$

(D) $\frac{11}{4}$

(E) 3

$f'(x)=\frac{f(3)-f(1)}{3-1}$

$2x=\frac{16-8}{2}$

$2x=4$

$x=2$

B 2. The weekly profit function of a company that produces unicorn widgets is $P(x)=-0.01x^2+3x-50000$, where x is the number of unicorn widgets made and sold. How many unicorn widgets must the company make and sell to maximize their profit?

(A) 300
(B) 150
(C) 600
(D) 60
(E) 200

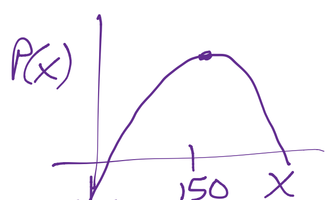
$P'(x)=-0.02x+3=0$

$0.02x=3$

$\frac{2}{100}x=3$

$\frac{1}{50}x=3$

$x=150$



D 3. A 2 meter by 2 meter square sheet of titanium alloy is used to make an open box by cutting squares of equal size from the four corners and folding up the sides. What size squares (side length in meters) should be cut to maximize the volume of the box?

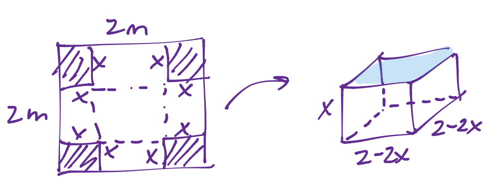
(A) 1

(B) $\frac{2}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$



$V=x(2-2x)^2,\ x\in[0,1]$

$V=x(4-8x+4x^2)$

$V=4x^3-8x^2+4x$

$V'=12x^2-16x+4=0$

$4(3x^2-4x+1)=0$

$4(3x-1)(x-1)=0$

$x=\frac{1}{3}$, $x=1$

maximizes Volume

minimizes Volume $=0$

C 4. A raccoon moves along an x-axis at a distance s away from the origin according to the equation $s(t)=4t^3-2t+1$, where $0\le t\le 10$. At what time is the raccoon farthest from the origin?

(A) 0
(B) 2
(C) 10

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$

Maximize $s(t)$

$s'(t)=12t^2-2=0$

$12t^2=2$

$t^2=\frac{1}{6}$

$t=\sqrt{\frac{1}{6}}$

$s(0)=1$

$s(\sqrt{\tfrac{1}{6}})=4(\tfrac{1}{6})\sqrt{\tfrac{1}{6}}-2\sqrt{\tfrac{1}{6}}+1=-\tfrac{4}{3}\sqrt{\tfrac{1}{6}}+1$

$s(10)=4000-20+1=3981$ ← MAX

A 5. The function $f(x)=x^{-1/9}$ has how many inflection points?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$f(x)=\frac{1}{\sqrt[9]{x}}$, $D_f: \{x \mid x\neq 0\}$

f has a VA @ $x=0$

$f'(x)=-\frac{1}{9}x^{-10/9}$

$f''(x)=\frac{10}{81}x^{-19/9}$

$f''(x)=\frac{10}{81\sqrt[9]{x^{19}}}=0$ Never → No p.i.'s

x	-1	$x=0$	1
f''	$-$		$+$

*$f(x)$ only changes concavity at $x=0$, but $x=0$ is NOT an inflection pt. since $x=0$ is NOT in the domain of $f(x)$.

B 6. The function $f(x)=x^4-8x^3$ has

(A) A relative maximum at $x=0$; no relative minimum
(B) No relative maximum; a relative minimum at $x=6$
(C) A relative maximum at $x=0$; a relative minimum at $x=6$
(D) A relative maximum at $x=0$; a relative minimum at $x=-6$ and $x=6$
(E) A relative minimum at $x=0$; a relative maximum at $x=-6$.

$f'(x)=4x^3-24x^2=0$

$\boxed{4x^2(x-6)}=0$

$x=0,\ x=6$

x	-1	0	1	6	7
f'	$-$		$-$		$+$

A 7. Using the linearization of $f(x)=\sqrt[3]{x}$ at $x=64$, how much less that 4 is the value of $\sqrt[3]{63}$?

(A) $\frac{1}{48}$
(B) $\frac{1}{16}$
(C) $\frac{1}{3}$
(D) $\frac{2}{3}$
(E) 1

$f(64)=4$
pt: $(64,4)$

$f'(x)=\frac{1}{3}x^{-2/3}$
$=\frac{1}{3\sqrt[3]{x^2}}$

$f'(64)=\frac{1}{3(\sqrt[3]{64})^2}$
$=\frac{1}{48}$

$m=\frac{1}{48}$

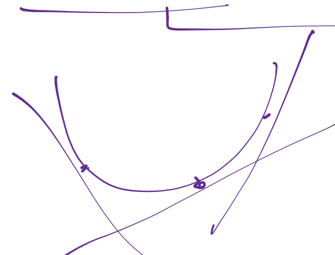
$\mathcal{L}(x)=4+\frac{1}{48}(x-64)$

$f(63)\approx\mathcal{L}(63)=4+\frac{1}{48}(-1)$
$=4-\frac{1}{48}$

D 8. If f is differentiable, we can use the line tangent to $f(x)$ at $x=c$ to approximate values of $f(x)$ near $x=c$. Suppose this method always underestimates the correct values. If so, in the neighborhood of $x=c$, f must be

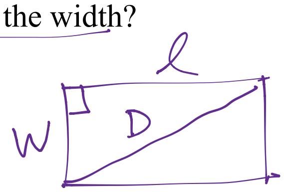
(A) positive
(B) increasing
(C) decreasing
(D) concave up
(E) concave down

→ tangent lines BELOW the curve, so the curve is CONCAVE UP



A 9. The length of a happy rectangle is increasing at 3 times the rate that its width is increasing. When the length is 12 feet and the width is 6 feet, the diagonal of the happy rectangle is increasing how many times faster that the width?

(A) $\frac{7}{\sqrt{5}}$
(B) $7\sqrt{5}$
(C) $\frac{5}{3\sqrt{5}}$
(D) $15\sqrt{5}$
(E) $6\sqrt{5}$



$\frac{dl}{dt}=3\frac{dW}{dt}$

$\boxed{l=12 \quad W=6}$

$\frac{dD}{dt}=?$

when $w=6$:
$6^2+12^2=D^2$
$D=\sqrt{36+144}$
$D=\sqrt{180}$
$D=\sqrt{9\cdot4\cdot5}$
$D=6\sqrt{5}$

$w^2+l^2=D^2$

$\frac{d}{dt}: 2w\frac{dw}{dt}+2l\frac{dl}{dt}=2D\frac{dD}{dt}$

$w\frac{dw}{dt}+l\frac{dl}{dt}=D\frac{dD}{dt}$

when $w=6$: $(6)\frac{dw}{dt}+12\left(3\frac{dw}{dt}\right)=6\sqrt{5}\frac{dD}{dt}$

$42\frac{dw}{dt}=6\sqrt{5}\frac{dD}{dt}$

$\frac{dD}{dt}=\frac{42}{6\sqrt{5}}\frac{dw}{dt}$

$\frac{dD}{dt}=\frac{7}{\sqrt{5}}\frac{dw}{dt}$

II. Free Response: Show all work in the space provided using correct notation. Include units on all final answers.

10. Non-alcoholic Wassail is draining at the rate of $\frac{\pi}{1000}\ \text{ft}^3/\text{sec}$ from the vertex the bottom of a HUGE German conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

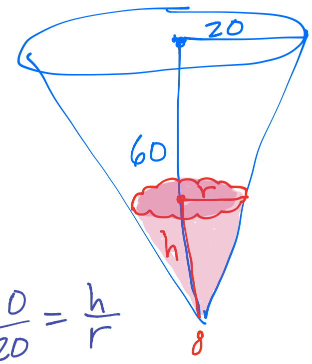
(a) Write an equation for the volume of the Wassail in the German tank, in cubic feet, in terms of its radius, in feet, at the surface of the Wassail.

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3} r^2 (3r)$

$\boxed{V = \pi r^3}$ ✓1

$\frac{dV}{dt} = -\frac{\pi}{1000}$



$\frac{60}{20} = \frac{h}{r}$

$\boxed{\begin{array}{l} h = 3r \\ r = \frac{1}{3}h \end{array}}$ ✓2

(b) At what rate, in feet per second, is the radius of the Wassail in the German tank changing when the radius is 10 feet?

$\frac{dr}{dt} = ?$ $\boxed{r = 10\text{ft}}$

$V = \pi r^3$

$\frac{d}{dt}: \frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$ ✓3

when $r=10$: $-\frac{\pi}{1000} = 3\pi(10^2)\frac{dr}{dt}$

$\frac{-\pi}{1000} = 300\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-1}{300000}$ ft/sec ✓4

(c) How fast is the height of the Wassail in the German tank changing, in feet per second, at the instant that the radius is 10 feet?

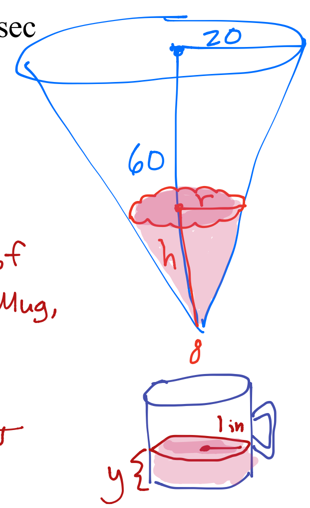
$\frac{dh}{dt} = ?$ $\qquad h = 3r$

$\frac{d}{dt}: \frac{dh}{dt} = 3\frac{dr}{dt}$ ✓5

when $r=10$: $\frac{dh}{dt} = 3\left(\frac{-1}{300{,}000}\right)$

$\frac{dh}{dt} = \frac{-1}{100{,}000}$ ft/sec ✓6

(d) If a teenaged unicorn calculus scholar places his cylindrical commemorative Wassail 2017 mug under the draining Wassail in the hope to imbibe the festive non-alcoholic beverage, how fast is the height of the Wassail in the commemorative mug rising (**in <u>inches</u> per second**) when the radius of the Wassail in the German tank is 10 feet? Assume the commemorative mug has a 2-inch diameter and is 3 inches tall with a cute, tiny little mug handle. NOTE: $\frac{\pi}{1000}\ \text{ft}^3/\text{sec} = 1.728\pi\ \text{in}^3/\text{sec}$



$V = \pi r^2 h$

$M = \pi(1\,\text{in})^2 y$

$M = \pi y$ ✓7

$\frac{d}{dt}: \frac{dM}{dt} = \pi\frac{dy}{dt}$ ✓8

when $r=10$: $1.728\pi = \pi\frac{dy}{dt}$

$\frac{dy}{dt} = 1.728$ in/sec ✓9

Let M be the volume of Wassail in the Mug, in in^3.

$\frac{dM}{dt} = 1.728\pi$

<-1> MAX for NO UNITS