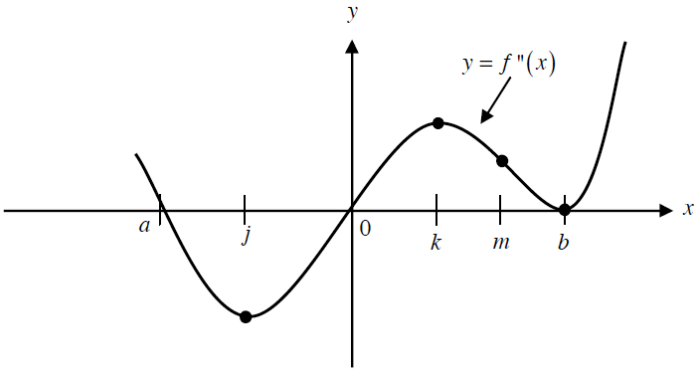
Name_____ Date_____ Period_____

AP Calculus AB/BC, TEST: 5.1 to 5.8

A

1. The second derivative of a function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown at right. For what values of x does the graph of f have a point of inflection?



(A) 0 and a only (B) 0 and m only
(C) j and b only (D) 0, a, and b (E) j, b, and k

B

2. Determine of the function $f(x) = x\sqrt{6-x}$ satisfies the hypothesis of the MVT on the interval $[0,6]$, and if it does, find all numbers c satisfying the conclusion of that theorem.

(A) $c = 5$ (B) $c = 4$ (C) $c = 3$ (D) $c = 2,\ 3$ (E) $c = 4,\ 5$ (F) hypothesis not satisfied

E

3. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x > -2$ (B) $x < -1$ (C) $x > -1$ (D) $x < 0$ (E) $x < -2$

D

4. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 1.4 (B) 1.3 (C) 0.7 (D) 0.6 (E) 0.4

C

5. A baseball diamond is a square with side 90 feet. If a batter hits the ball and runs towards first base with a speed of 25 ft/sec, at what speed is his distance from second base decreasing when he is two thirds of the way to first base?

(A) $2\sqrt{10}$ ft/sec (B) $3\sqrt{5}$ ft/sec (C) $\frac{5}{2}\sqrt{10}$ ft/sec (D) $\frac{3}{2}\sqrt{10}$ ft/sec (E) $4\sqrt{5}$ ft/sec

G

6. Find the values of x at which the graph of $y = x^2 - 4\cos x$ changes concavity on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(A) $x = \frac{\pi}{3}$ (B) $x = -\frac{\pi}{3}$ (C) $x = -\frac{\pi}{3},\ \frac{\pi}{3}$ (D) $x = -\frac{\pi}{6},\ \frac{\pi}{6}$

(E) $x = \frac{\pi}{6}$ (F) $x = -\frac{\pi}{6}$ (G) there are no values of x

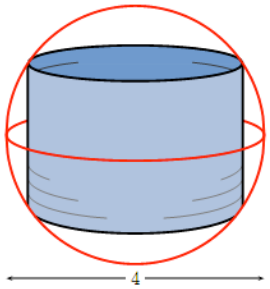
E

7. Let f be the function with derivative given by $f'(x) = 2x^2 - 15x + 25$. How many local extrema does f have on the interval $2 < x < 4$?

(A) Five (B) Four (C) Three (D) Two (E) One

A

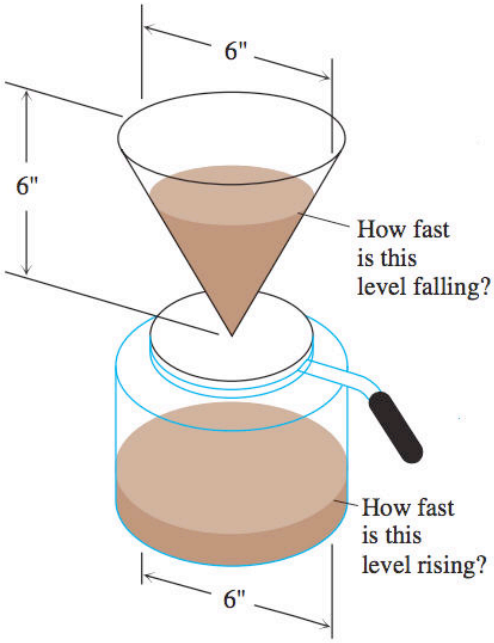
8. A right circular cylinder is inscribed in a sphere with **diameter** 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.



(A) $A = 8\pi$ cm^2 (B) $A = 8$ cm^2 (C) $A = 16$ cm^2

(D) $A = 16\pi$ cm^2 (E) $A = 2$ cm^2

Part II: Free Response. Do all work below the line. Label each part. Notation, Notation, Notation. Include units in ALL of your final answers.



9. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 8 in^3 / min . The dimensions of the filter and coffeepot are indicated in the diagram at right. Note: 6″ = 6 inches .

① A
② B
③ E
④ D
⑤ C
⑥ G
⑦ E
⑧ A

(a) Using similar triangles, find an equation relating the height, ***h***, of the coffee in the cone in terms of the radius, ***r***, of the coffee in the cone.

(b) Write a simplified equation for the volume, ***V***, of the coffee in the cone in terms of the height, ***h***, of coffee in the cone. (get rid of the *r* variable!)

(c) How much coffee, in cubic inches, is in the cone when the coffee in the cone is 4 inches deep?

(d) How fast is the level, ***h***, in the cone falling when the coffee in the cone is 4 inches deep?

(e) How fast is the depth level, ***y***, in the pot rising when the coffee in the cone is 4 inches deep?

(f) Do you prefer hot coffee or iced coffee? Precalculus or Calculus?

(a) $\frac{3}{6} = \frac{r}{h}$

$\frac{1}{2}h = r$

$h = 2r$

(b) $V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12} h^3$

(c) $h = 4$

$V(4) = \frac{\pi}{12}(4^3) = \frac{64\pi}{12}$ in^3/min

or $\frac{16\pi}{3}$

(d) $\frac{dh}{dt} = ?, \frac{dV}{dt} = -8$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

when h=5: $-8 = \frac{\pi}{4}(4^2)\frac{dh}{dt}$

$-8 = \frac{16\pi}{4}\frac{dh}{dt} = 4\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-8}{4\pi}$ in/min or $-\frac{2}{\pi}$

(e) $\frac{dy}{dt} = ?$ $V = \pi(3^2)y$

$V = 9\pi y$

$\frac{dV}{dt} = 9\pi\frac{dy}{dt}$

when h=5: $8 = 9\pi\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{8}{9\pi}$ in/min

(f) Hot Calculus

8 checks

16 total checks