

AP Calculus AB/BC, TEST: 3.1 to 3.8

\_\_\_\_\_ 1. Find the values of  $x$  at which the graph of  $y = x^2 - 4 \cos x$  changes concavity on  $\left(-\frac{f}{2}, \frac{f}{2}\right)$ .

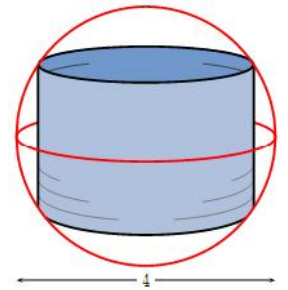
- (A)  $x = \frac{f}{6}$     (B)  $x = -\frac{f}{3}$     (C) there are no values of  $x$     (D)  $x = -\frac{f}{3}, \frac{f}{3}$   
 (E)  $x = \frac{f}{3}$     (F)  $x = -\frac{f}{6}, \frac{f}{6}$     (G)  $x = -\frac{f}{6}$

\_\_\_\_\_ 2. The function  $f$  is twice differentiable with  $f(2) = 1$ ,  $f'(2) = 4$ , and  $f''(2) = 3$ . What is the value of the approximation of  $f(1.9)$  using the line tangent to the graph of  $f$  at  $x = 2$ ?

- (A) 0.4    (B) 0.6    (C) 0.7    (D) 1.3    (E) 1.4

\_\_\_\_\_ 3. A right circular cylinder is inscribed in a sphere with **diameter** 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.

- (A)  $A = 8 \text{ cm}^2$     (B)  $A = 16 \text{ cm}^2$     (C)  $A = 16f \text{ cm}^2$   
 (D)  $A = 2 \text{ cm}^2$     (E)  $A = 8f \text{ cm}^2$



\_\_\_\_\_ 4. Let  $f$  be the function given by  $f(x) = 2xe^x$ . The graph of  $f$  is concave down when

- (A)  $x < -2$     (B)  $x > -2$     (C)  $x < -1$     (D)  $x > -1$     (E)  $x < 0$

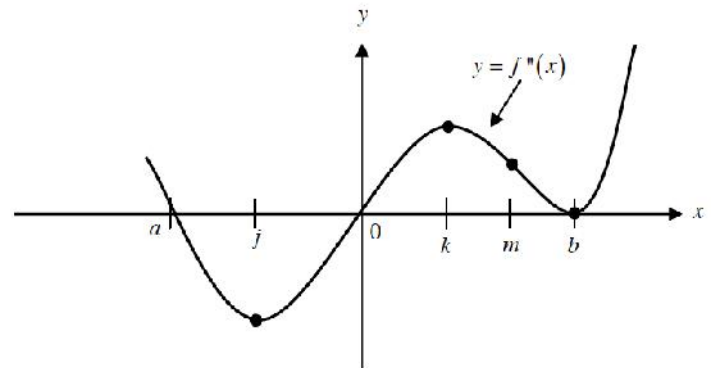
\_\_\_\_\_ 5. A baseball diamond is a square with side 90 feet. If a batter hits the ball and runs towards first base with a speed of 25 ft/sec, at what speed is his distance from second base decreasing when he is two thirds of the way to first base?

- (A)  $\frac{5}{2}\sqrt{10}$  ft/sec    (B)  $\frac{3}{2}\sqrt{10}$  ft/sec    (C)  $4\sqrt{5}$  ft/sec    (D)  $2\sqrt{10}$  ft/sec    (E)  $3\sqrt{5}$  ft/sec

\_\_\_\_\_ 6. Let  $f$  be the function with derivative given by  $f'(x) = 2x^2 - 15x + 25$ . How many local extrema does  $f$  have on the interval  $2 < x < 4$ ?

- (A) One    (B) Two    (C) Three    (D) Four    (E) Five

\_\_\_\_\_ 7. The second derivative of a function  $f$  is given by  $f''(x) = x(x-a)(x-b)^2$ . The graph of  $f''$  is shown at right. For what values of  $x$  does the graph of  $f$  have a point of inflection?



- (A) 0 and  $a$  only    (B) 0 and  $m$  only  
 (C)  $j$  and  $b$  only    (D) 0,  $a$ , and  $b$     (E)  $j$ ,  $b$ , and  $k$

\_\_\_\_\_ 8. Determine if the function

$$f(x) = x\sqrt{6-x}$$

satisfies the hypothesis of the MVT on the interval  $[0, 6]$ , and if it does, find all numbers  $c$  satisfying the conclusion of that theorem.

- (A)  $c = 2, 3$     (B)  $c = 4, 5$     (C)  $c = 5$     (D)  $c = 3$     (E)  $c = 4$     (F) hypothesis not satisfied





Part II: Free Response. Do all work below the line. Label each part. Notation, Notation, Notation. Include units in ALL of your final answers.

9. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of  $10 \text{ in}^3 / \text{min}$ . The dimensions of the filter and coffeepot are indicated in the diagram at right.  
Note:  $6'' = 6 \text{ inches}$ .

- Using similar triangles, find an equation relating the height,  $h$ , of the coffee in the cone in terms of the radius,  $r$ , of the coffee in the cone.
- Write a simplified equation for the volume,  $V$ , of the coffee in the cone in terms of the height,  $h$ , of coffee in the cone. (get rid of the  $r$  variable!)
- How much coffee, in cubic inches, is in the cone when the coffee in the cone is 5 inches deep?
- How fast is the level,  $h$ , in the cone falling when the coffee in the cone is 5 inches deep?
- How fast is the depth level,  $y$ , in the pot rising when the coffee in the cone is 5 inches deep?
- Do you prefer hot coffee or iced coffee? Precalculus or Calculus?

