Name KEY Date__________________ Period________

AP Calculus AB/BC, TEST: 5.1 to 5.8

C 1. Find the values of x at which the graph of $y = x^2 - 4\cos x$ changes concavity on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(A) $x = \frac{\pi}{6}$ (B) $x = -\frac{\pi}{3}$ (C) there are no values of x (D) $x = -\frac{\pi}{3}, \frac{\pi}{3}$

(E) $x = \frac{\pi}{3}$ (F) $x = -\frac{\pi}{6}, \frac{\pi}{6}$ (G) $x = -\frac{\pi}{6}$

B 2. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

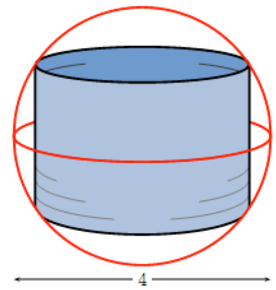
(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$L = 1 + 4(x-2) = 1 + 4(-.1) = 1 - .4 = .6$

E 3. A right circular cylinder is inscribed in a sphere with **diameter** 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.

(A) $A = 8\text{ cm}^2$ (B) $A = 16\text{ cm}^2$ (C) $A = 16\pi\text{ cm}^2$

(D) $A = 2\text{ cm}^2$ (E) $A = 8\pi\text{ cm}^2$



A 4. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$y' = 2e^x + 2xe^x$, $y'' = 2e^x + 2e^x + 2xe^x = 2e^x(2+x)$, $x = -2$ y'': -3 | -2 | 0: $-$ $+$

A 5. A baseball diamond is a square with side 90 feet. If a batter hits the ball and runs towards first base with a speed of 25 ft/sec, at what speed is his distance from second base decreasing when he is two thirds of the way to first base?

$y = 30$, $x = 30\sqrt{10}$; $30^2 + 90^2 = x^2$, $900 + 8100$, $x^2 = 9000 = 900 \cdot 10$, $x = 30\sqrt{10}$

(A) $\frac{5}{2}\sqrt{10}$ ft/sec (B) $\frac{3}{2}\sqrt{10}$ ft/sec (C) $4\sqrt{5}$ ft/sec (D) $2\sqrt{10}$ ft/sec (E) $3\sqrt{5}$ ft/sec

$x^2 + y^2 = 90^2$
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$
$\frac{dy}{dt} = -25$ || $30\sqrt{10} \cdot \frac{dx}{dt} + 30\left(-\frac{25}{1}\right) = 0$
$\frac{dx}{dt} = \frac{750}{30\sqrt{10}} = \frac{25\sqrt{10}}{10} = \frac{5\sqrt{10}}{2}$

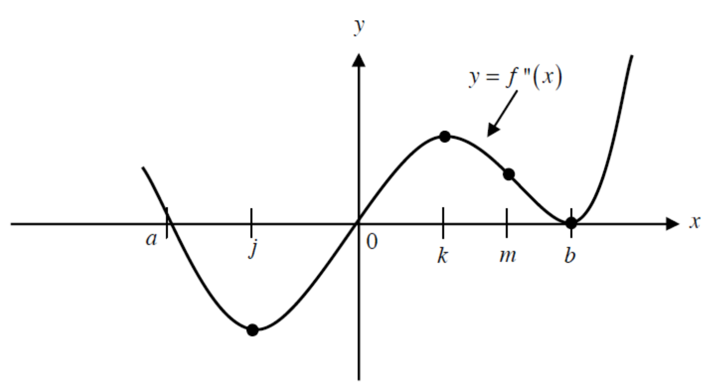
A 6. Let f be the function with derivative given by $f'(x) = 2x^2 - 15x + 25$. How many local extrema does f have on the interval $2 < x < 4$?

$= (2x-5)(x-5)$ $x = \frac{5}{2}, x = 5$
$2.5, 5$

(A) One (B) Two (C) Three (D) Four (E) Five

A 7. The second derivative of a function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown at right. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only
(C) j and b only (D) 0, a, and b (E) j, b, and k



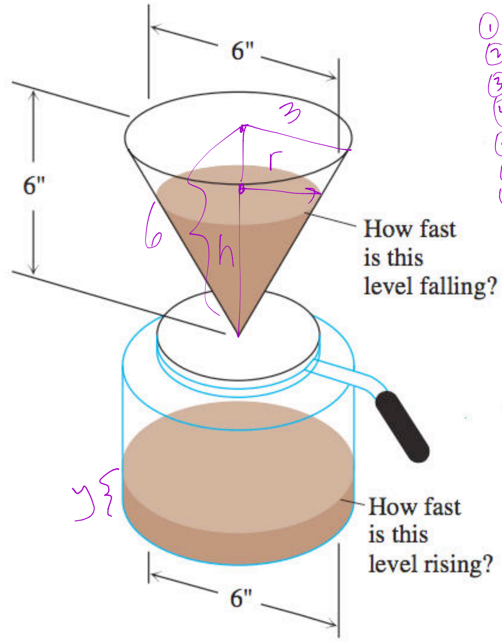
E 8. Determine of the function $f(x) = x\sqrt{6-x}$ satisfies the hypothesis of the MVT on the interval $[0,6]$, and if it does, find all numbers c satisfying the conclusion of that theorem.

(A) $c = 2, 3$ (B) $c = 4, 5$ (C) $c = 5$ (D) $c = 3$ (E) $c = 4$ (F) hypothesis not satisfied

$(6-x)^{1/2} + x\left(\frac{1}{2}\right)(6-x)^{-1/2}(-1)$
$(6-x)^{-1/2}\left[6 - x - \frac{1}{2}x\right]$
$\frac{6 - \frac{3}{2}x}{\sqrt{6-x}}$
$\frac{12-3x}{2\sqrt{6-x}} = 0$ $12 - 3x = 0$ $\boxed{x = 4}$

Part II: Free Response. Do all work below the line. Label each part. Notation, Notation, Notation. Include units in ALL of your final answers.

9. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of $10\ \text{in}^3/\text{min}$. The dimensions of the filter and coffeepot are indicated in the diagram at right. Note: $6'' = 6$ inches.



① C
② B
③ E
④ A
⑤ A
⑥ A
⑦ A
⑧ E

(a) Using similar triangles, find an equation relating the height, ***h***, of the coffee in the cone in terms of the radius, ***r***, of the coffee in the cone.

(b) Write a simplified equation for the volume, ***V***, of the coffee in the cone in terms of the height, ***h***, of coffee in the cone. (get rid of the *r* variable!)

(c) How much coffee, in cubic inches, is in the cone when the coffee in the cone is 5 inches deep?

(d) How fast is the level, ***h***, in the cone falling when the coffee in the cone is 5 inches deep?

(e) How fast is the depth level, ***y***, in the pot rising when the coffee in the cone is 5 inches deep?

(f) Do you prefer hot coffee or iced coffee? Precalculus or Calculus?

(a) $\frac{3}{6} = \frac{r}{h}$

$\boxed{\begin{gathered}\tfrac{1}{2}h = r \\ h = 2r\end{gathered}}$

(b) $V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$\boxed{V = \frac{\pi}{12} h^3}$

(c) $\boxed{h = 5}$

$V(5) = \boxed{\frac{\pi}{12}(5^3) = \frac{125\pi}{12}}\ \text{in}^3/\text{min}$

(d) $\frac{dh}{dt} = ?$, $\frac{dV}{dt} = -10$

$V = \frac{\pi}{12} h^3$

$\boxed{\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}}$

when $h=5$: $-10 = \frac{\pi}{4}(5^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{(-10)(4)}{25\pi}$

$\frac{dh}{dt} = \boxed{\frac{-8}{5\pi}}\ \text{in}/\text{min}$

(e) $\frac{dy}{dt} = ?$ $V = \pi(3^2)y$

$\boxed{V = 9\pi y}$

$\frac{dV}{dt} = 9\pi \frac{dy}{dt}$

when $h=5$: $10 = 9\pi \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = \frac{10}{9\pi}}\ \text{in}/\text{min}$

(f) Hot Calculus

16 total Checks

8 checks

1973 AB 6

A manufacturer finds it costs him $x^2 + 5x + 7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x - 3)$ dollars on his total production. If the price he receives is \$13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?

1976 AB 4

a. A point moves on the hyperbola $3x^2 - y^2 = 23$ so that its y-coordinate is increasing at a constant rate of 4 units per second. How fast is the x-coordinate changing when
$x = 4$?

b. For what values of k will the line $2x + 9y + k = 0$ be normal to the hyperbola $3x^2 - y^2 = 23$?

1982 AB 4

A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.