Name KEY ____________________ Date ____________________ Period ________

AP Calculus AB/BC
TEST: 5.1 to 5.8, **Calculator OK**

$f'(x) = \frac{50\cos\sqrt{50}}{50}$

__C__ 1. How many values of c satisfy the Mean Value Theorem for the equation $f(x) = x\cos\left(\sqrt{x}\right)$, $0 \le x \le 50$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

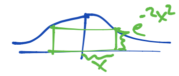
__B__ 2. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$y = L(x) = 1 + 4(x-2)$
$f(1.9) \approx L(1.9) = 1 + 4(1.9-2) = 0.6$

__E__ 3. A rectangle has one side on the x-axis and the upper two vertices on the graph of $y = e^{-2x^2}$. Give a decimal approximation for the maximum possible area for this rectangle.

(A) 1.649 (B) 1 (C) 2.031 (D) 0.545 (E) 0.606

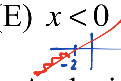


e^{-2x^2} $A = 2xe^{-2x^2}$
$A_{max} = 0.606$

__A__ 4. Let f be the function given by $f(x) = 2xe^{x}$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$f' = 2e^x + 2xe^x$
$f'' = 2e^x + 2e^x + 2xe^x$
$f'' = 4e^x + 2xe^x = e^x(4+2x)$
$4 + 2x = 0,\ x = -2$



__C__ 5. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere?

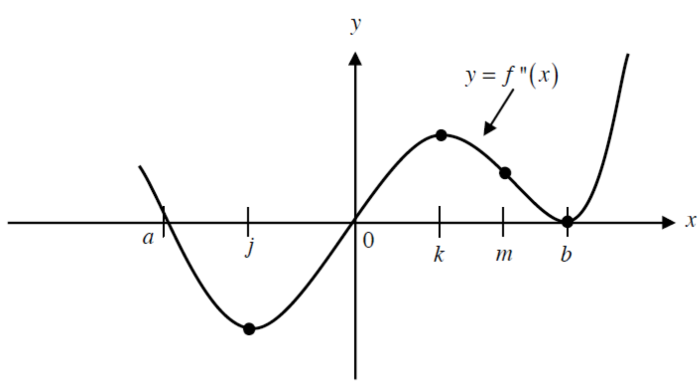
(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

$\frac{dr}{dt} = -2$, $\boxed{r=3}$ $A = 4\pi r^2$
$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$
$\frac{dA}{dt} = 8\pi(3)(-2) = -48\pi$

__D__ 6. Let f be the function with derivative given by $f'(x) = \sin\left(x^2+1\right)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

__A__ 7. The second derivative of a function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown at right. For what values of x does the graph of f have a point of inflection?



(A) 0 and a only (B) 0 and m only
(C) j and b only (D) 0, a, and b (E) j, b, and k

__B__ 8. If $f(x) = 3x^2 + x$, $x = 2$, and $dx = 0.002$, find dy.

(A) 0.02 (B) 0.026 (C) 0.028 (D) 0.014 (E) 0.26

$dy = (6x+1)dx$
$dy\big|_{x=2} = (13)(0.002) = 0.026$

Part II: Free Response. Do all work below the line. Label each part. Notation, Notation, Notation.

10. (1984-AB5) The volume V of a cone is increasing at the rate of 28π cubic inches per second. At the instant when the radius r on the cone is 3 inches, its volume is 12π cubic inches, and the radius is increasing at $\frac{1}{2}$ inches per second.

(a) At the instant when the radius of the cone is 3 inches, what is the rate of change of the area of the base?
(b) At the instant when the radius of the cone is 3 inches, what is the rate of change of its height h?
(c) At the instant when the radius of the cone is 3 inches, what is the instantaneous rate of change of the area of its base with respect to its height h?

$\boxed{\frac{dV}{dt} = 28\pi}$

$\boxed{\frac{dr}{dt} = \frac{1}{2}}$

$V = \frac{\pi}{3} r^2 h$, when $\boxed{r = 3, V = 12\pi \quad h = 4}$

$12\pi = \frac{\pi}{3}(3^2)h$

$h = \frac{12\pi}{3\pi} = \boxed{4 = h}$

(a) $\frac{dA}{dt} = ?$ $A = \pi r^2$

$\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$

when $r = 3$:

$\frac{dA}{dt} = 2\pi(3)\left(\frac{1}{2}\right)$

$= \boxed{3\pi}$ in²/sec

(b) $V = \frac{\pi}{3} r^2 h$

$\boxed{\frac{dV}{dt} = \frac{2\pi}{3} r \frac{dr}{dt} h + \frac{\pi}{3} r^2 \frac{dh}{dt}}$

when $r = 3$:

$28\pi = \frac{2\pi}{3}(3)\left(\frac{1}{2}\right)(4) + \frac{\pi}{3}(3^2)\frac{dh}{dt}$

$28\pi = 4\pi + 3\pi \frac{dh}{dt}$

$3\pi \frac{dh}{dt} = 24\pi$

$\boxed{\frac{dh}{dt} = 8}$ in/sec

(c) $\frac{dA}{dh} = \frac{dA/dt}{dh/dt} = \boxed{\frac{3\pi}{8}}$ in²/sec

OR

$A = \pi r^2$

$\frac{dA}{dh} = 2\pi r \frac{dr}{dh}$

$\frac{dA}{dh} = 2\pi r \frac{dr/dt}{dh/dt}$

when $r = 3$: $\frac{dA}{dt} = 2\pi(3)\left(\frac{1/2}{8}\right) = \frac{3\pi}{8}$

units 1 ✓

1973 AB 6

A manufacturer finds it costs him $x^2 + 5x + 7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x - 3)$ dollars on his total production. If the price he receives is $13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?

1976 AB 4

a. A point moves on the hyperbola $3x^2 - y^2 = 23$ so that its y-coordinate is increasing at a constant rate of 4 units per second. How fast is the x-coordinate changing when
$x = 4$?

b. For what values of k will the line $2x + 9y + k = 0$ be normal to the hyperbola $3x^2 - y^2 = 23$?

1982 AB 4

A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.