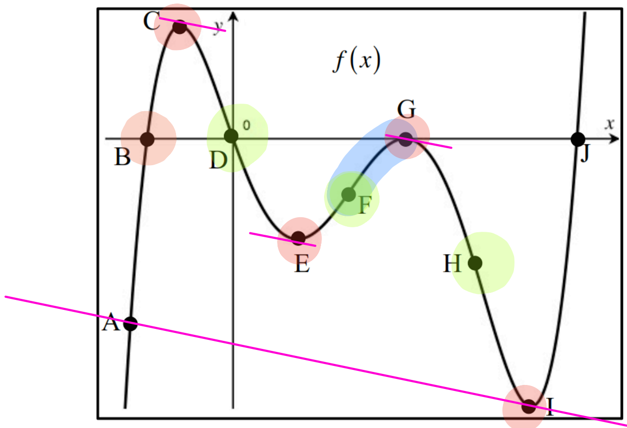
18 total pts
9MC
9FR

Name KEY Date The day before Tomorrow Santa's Favorite Exercise YOGA CORPSE POSE

AP Calculus TEST: 3.1-3.7, NO CALCULATOR

Part I: Multiple Choice—Put the CAPITAL letter of the correct response in the blank



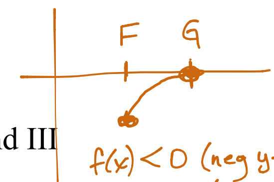
Secant line is parallel to tangent line on (A, I)
Question 4: MVT
OPEN INTERVAL

The graph of the function $y = f(x)$ is shown above. Use the graph to answer questions 1 – 5.
*Assume all CAPITAL letters represent the x-values of the indicated points.

D 1. On the interval (F, G), which of the following is true?

I. $f(x) < 0$ ✓ II. $f'(x) > 0$ ✓ III. $f''(x) < 0$ ✓

(A) I only (B) I and II only (C) I and III only (D) I, II, and III



$f(x) < 0$ (neg y-values)
$f'(x) > 0$ (increasing)
$f''(x) < 0$ (concave down)

C 2. How many critical values does $y = f(x)$ have on the interval shown?

CV: $f' = $ DNE or $f' = 0$
(A) 2 (B) 3 (C) 4 (D) 5

$f' = $ DNE → 0; $f' = 0$ → 4 at $x = C, E, G, I$

C 3. How many inflection values does $y = f(x)$ have on the interval shown?

concave up to down or concave down to up
(A) 1 (B) 2 (C) 3 (D) 4

3, at $x = D, F, H$

D 4. How many values satisfy the MVT for $y = f(x)$ on the interval $[A, I]$?

(A) 0 (B) 1 (C) 2 (D) 3

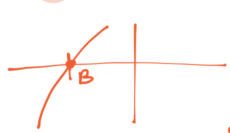
3, at the points with the small, pink tangent lines.

D 5. Which of the following inequalities is correct?

(A) $f(B) < f'(B) < f''(B)$ (B) $f'(B) < f(B) < f''(B)$

(C) $f''(B) < f'(B) < f(B)$ (D) $f''(B) < f(B) < f'(B)$

At $x = B$



$f(B) = 0$ (x-int)
$f'(B) > 0$ (increasing)
$f''(B) < 0$ (concave down)
So, neg < 0 < pos → $f''(B) < f(B) < f'(B)$

will be 1 of these 4

Horizontal tangent

No curvature at $x=a$, So, cannot be A or B!

$x=a$ is a p.i.v.

$f''(A)<0$ cc down $f''(B)>0$ cc up

A ___ 6. A function $y=f(x)$ has the properties that $f'(a)=0$ and $f''(a)=0$. Which one of the following statements must be true? for All $f(x)$ at all $x=a$

So, the graph MUST look like C, D, E, F

√ (A) The graph of $y=f(x)$ has a horizontal tangent at $(a, f(a))$.

Not always (B) $(a, f(a))$ is a point of inflection (No! Let $y=x^4$, $y'=4x^3=0$ at $x=0 \to$ cv. , $y''=12x^2=0$ at $x=0 \to$ p.i.v., but $y''>0$ on either side of $x=0 \to$ No curvature change $\to$ no inflection value

Not always (C) $(a, f(a))$ is either a local maximum or a local minimum point., No, see C & D above

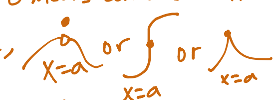
Never (D) f may be discontinuous at $x=a$. if $f'(a)=0$, f is differentiable at $x=a$, so it is also continuous at $x=a$

ONLY (A) is Always true!

A ___ 7. Given any given function $y=f(x)$, ***how many*** of the following statements must be true?

Never ~~I~~. If $f''(a)<0$, then the graph of $y=f(x)$ is concave up at $x=a$., No, $f''<0$ means concave down.

Not always ~~II~~. If $f'(a)$ does not exist, then $x=a$ is not in the domain of $y=f(x)$. false,



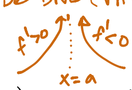
Never ~~III~~. If $f'(a)=0$ and $f''(a)>0$, then $f(a)$ is a local max. False, by 2nd Deriv test CC up at a cv ∪ means local MIN.

Not always ~~IV~~. If $f'(a)=DNE$ and $f'(x)$ changes from neg to pos at $x=a$, then $f(a)$ is a local min.

So, NONE are true

(A) 0 (B) 1 (C) 2 (D) 3

False $f(a)$ could be DNE (VA @ $x=a$)



C ___ 8. The function $y=f(x)$ is twice differentiable with $f(3)=-2$, $f'(3)=\frac{1}{2}$, and $f''(3)=1$. What is the value of the approximation of $f(4)$ using the line tangent to the graph of $y=f(x)$ at $x=3$? extra info

(A) -1.9 (B) -1.7 (C) -1.5 (D) -1.3

pt: $(3,-2)$

slope: $\frac{1}{2}$

$\mathcal{L}(x)=-2+\frac{1}{2}(x-3)$

$f(4)\approx\mathcal{L}(4)=-2+\frac{1}{2}(4-3)$

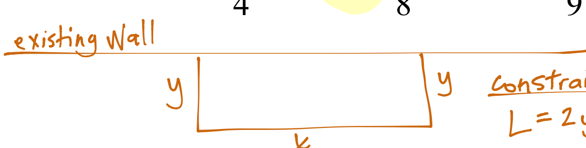
$=-2+\frac{1}{2}$

$=-\frac{4}{2}+\frac{1}{2}$

$=-\frac{3}{2}=-1.5$

B ___ 9. Given L feet of fencing, what is the maximum number of square feet that can be enclosed if the fencing is used to make three sides of a rectangular pen, using an existing wall as the fourth side?

(A) $\frac{L^2}{4}$ (B) $\frac{L^2}{8}$ (C) $\frac{L^2}{9}$ (D) $\frac{L^2}{16}$



Constraint

$L=2y+x$, for some constant L.

So, $x=L-2y$

Maximize Area, A

$A=xy$

Subbing: $A=(L-2y)y$, $y>0$

$A=Ly-2y^2$, L is a constant

$A'(y)=\frac{dA}{dy}=L-4y=0$

$L=4y$

$y=\frac{L}{4}$

So, Max area is $A\left(\frac{L}{4}\right)=\left(L-2\left(\frac{L}{4}\right)\right)\left(\frac{L}{4}\right)$

$=\left(L-\frac{L}{2}\right)\left(\frac{L}{4}\right)$

$=\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)$

$=\frac{L^2}{8}$

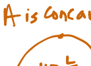
If you're interested in the Justification...

* Justify by Global 2nd Deriv Test

$A''(y)=-4<0$ for all y, so

A is concave down for all y

So, $y=\frac{L}{4}$ Globally Maximizes Area



Part II: Free Response—Do the work in the space provided. Use proper notation. f is continuous $\forall x \in \mathbb{R}$

10. (1981 AB3) Let f be the function defined by $f(x) = 12x^{2/3} - 4x$. $= 12\sqrt[3]{x^2} - 4x$ $\quad D_f : \mathbb{R}$

(a) Find the intervals on which f is increasing.

$f(x) = 12x^{2/3} - 4x$

cusp/pt at $x=0$!!

√1 $f'(x) = 8x^{-1/3} - 4$

$= \dfrac{8}{\sqrt[3]{x}} - 4$

$= \dfrac{8}{\sqrt[3]{x}} - \dfrac{4\sqrt[3]{x}}{\sqrt[3]{x}}$

$= \dfrac{8 - 4\sqrt[3]{x}}{\sqrt[3]{x}}$

$f'(x) = \dfrac{4(2 - \sqrt[3]{x})}{\sqrt[3]{x}}$

$f'(x) = DNE$ $\quad \sqrt[3]{x} = 0$ $\quad \boxed{x = 0}$

So, at $x=0$ f is 1 of

$f'(x) = 0$ $\quad 4(2 - \sqrt[3]{x}) = 0$ $\quad \sqrt[3]{x} = 2$ $\quad \boxed{x = 8}$ √2 BOTH NUMBERS

So, at $x=8$ f is 1 of

$f'(x) = \dfrac{4(2 - \sqrt[3]{x})}{\sqrt[3]{x}}$

Number Line Chart for $f'(x)$

x	-1	0	1	8	27
$f'(x)$	$-$		$+$		$-$

$\boxed{f \text{ is increasing on } x \in (0, 8)}$ √3

(since on this interval, $f'(x) > 0$)

(b) Find the x- and y- coordinates of all relative maximum points. Justify.

Number Line Chart for $f'(x)$

x	-1	cusp $x=0$	1	Horiz Tan $x=8$	27
$f'(x)$	$-$		$+$		$-$

(No discons of f to put in chart)

*I chose perfect cubes on the interval based on $f'(x)$

$f'(x) = \dfrac{4(2 - \sqrt[3]{x})}{\sqrt[3]{x}}$

$f'(-1) < 0$
$f'(1) > 0$
$f'(27) < 0$

$f(x) = 12x^{2/3} - 4x$

$f(0) = 0 - 0 = 0$

$f(8) = 12(\sqrt[3]{8})^2 - 4(8)$
$= 12(4) - 32$
$= 48 - 32$
$= 16$

√4 or $(8, 16)$

$\boxed{f \text{ has a local max of } y = 16 \text{ at } x = 8}$

$\boxed{\text{Since } f' \text{ changes from positive to negative at } x = 8}$

(First Deriv Test) √5 Justification for OR

or $f'(x) = 8x^{-1/3} - 4$

$f''(x) = -\dfrac{8}{3}x^{-4/3}$

$f''(x) = \dfrac{-8}{3\sqrt[3]{x^4}}$

$f''(8) = \dfrac{-8}{48} < 0$

CV

So, f has a local max of $y = 16$ at $x = 8$.

(Second Deriv Test)

(c) Find the x- and y- coordinates of all relative minimum points. Justify.

from part (b)

√6 or at $(0, 0)$

$\boxed{f \text{ has a local min of } y = 0 \text{ at } x = 0}$

$\boxed{\text{Since } f' \text{ changes from negative to positive at } x = 0.}$

√7 Justification

(First Deriv Test)

Second Deriv Test cannot be used since

$f''(0) = \dfrac{-8}{0}$ (undefined

(d) Find the intervals on which f is concave downward. want $f''(x)<0$

Finding f''

Method 1
easier,
less fun,
safer

$$f'(x) = 8x^{-1/3} - 4$$

$$f''(x) = -\frac{8}{3}x^{-4/3}$$

$$f''(x) = \frac{-8}{3\sqrt[3]{x^4}}$$

Method 2
More challenging,
more fun,
more perilous

$$f'(x) = \frac{4(2-\sqrt[3]{x})}{\sqrt[3]{x}}$$

$$f'(x) = \frac{8-4x^{1/3}}{x^{1/3}}$$

Quotient Rule:

$$f''(x) = \frac{(x^{1/3})(-\frac{4}{3}x^{-2/3}) - (8-4x^{1/3})(\frac{1}{3}x^{-2/3})}{(x^{1/3})^2}$$

these terms cancel!!!

$$= \frac{-\frac{4}{3}x^{-1/3} - \frac{8}{3}x^{-2/3} + \frac{4}{3}x^{-1/3}}{x^{2/3}}$$

$$= -\frac{8}{3}x^{-2/3}\cdot x^{-2/3}$$

$$= -\frac{8}{3}x^{-4/3}$$

Method 1: find p.i.v.s & test intervals.

$$f''(x) = \frac{-8}{3\sqrt[3]{x^4}}$$

$f''(x) = \text{DNE}$: $3\sqrt[3]{x^4} = 0$, $x = 0$

$f''(x) = 0$ when $-8 = 0$ (No solutions)

Number Line Chart for f''

x	-1	0	1
f''	$-$	$f''=\text{DNE}$	$-$

So, f is concave down for all $x \neq 0$ $\sqrt{8}$

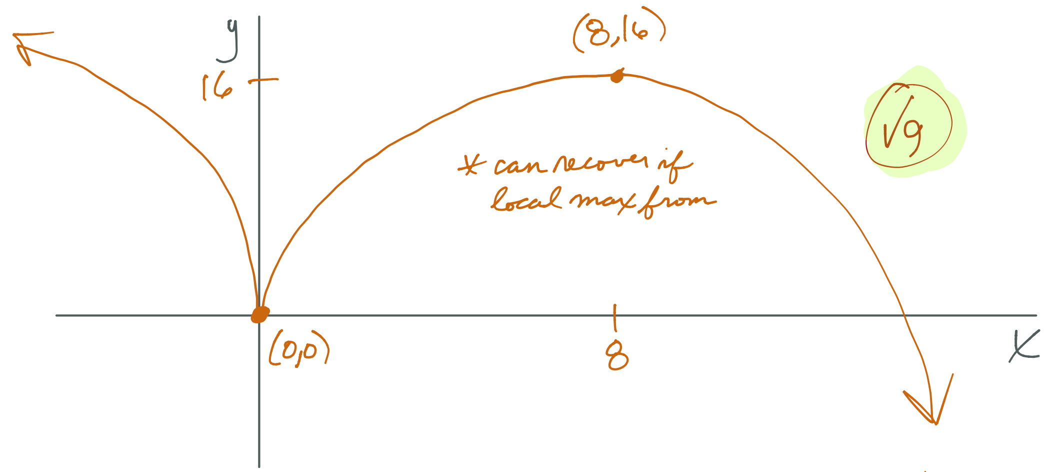
or ... on $(-\infty,0)\cup(0,\infty)$

Method 2: Analysis

Notice that $f''(x) < 0$ for all $x \neq 0$ ($f''(0) = \text{DNE}$)

So, f is concave down for all $x \neq 0$

(e) Using the information found in the parts above, sketch the graph of f.



$\sqrt{9}$

Something like that.

9 pts FR