10 M.C. + 10 F.R.

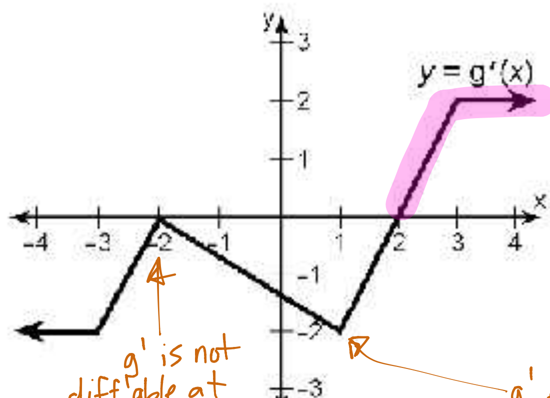
Name KEY Date 20 pts total Brand of Facial Tissues__________

TEST: 3.1-3.6, NO CALCULATOR

Part I: Multiple Choice: Put the letter in the letter place.

C 1. The graph of the derivative, $g'(x)$, of a function $g(x)$ is shown below

• y-values of g' are slopes of g
• slopes of g' are concavity values of g.



g' is not diff'able at $x=-2 \rightarrow g''(-2) = \text{DNE}$

g' exists $\forall x \in [-4,4]$, so g is diff'able → continuous on $[-4,4]$

Which of the following must be true about the function $g(x)$ on the interval $[-4,4]$?

I. $g(x)$ is increasing for $x>2$ **only** ✓ since $g'>0$ on $(2,4]$

II. $g(x)$ is not differentiable at four points ✗ (true of g', not g)

III. $g(x)$ is concave down for $-2<x<1$ ✓ since slopes of g' are neg on $(-2,1)$

(A) I, II, and III (B) I only (C) I and III only (D) I and II only (E) II only

D 2. On what open intervals is $f(x)=\dfrac{2x-3}{x^2}$ increasing?, $x \neq 0$ (f has VA @ x=0)

(A) $(3,\infty)$ (B) $(0,\infty)$ (C) $(-\infty,3)$ (D) $(0,3)$ (E) $(-\infty,-3)$

$f' = \dfrac{x^2(2)-(2x-3)(2x)}{(x^2)^2}$

$f' = \dfrac{2x[x-2x+3]}{x^4}$

$f' = \dfrac{2x(3-x)}{x^4}$

$f'=0$
$x=3$
VA @ $x=0$

x	-1	0	1	3	4
f'	−		+		−

f is inc on $(0,3)$, since $f'>0$ on $(0,3)$

A 3. If $\lim\limits_{h\to 0}\dfrac{f(-3+h)-f(-3)}{h}=2.718$, then the graph of $f(x)$ at $x=-3$ is

(A) increasing (B) concave up (C) decreasing (D) stationary (E) concave down

limit def of $f'(-3)$ (modified form)
So, $f'(-3)=2.718>0$,
So, f is increasing at $x=-3$

B 4. On the interval $[0,\pi]$, the graph of $f(x)=\frac{1}{2}x+\sin x$ has a critical value at $x=$

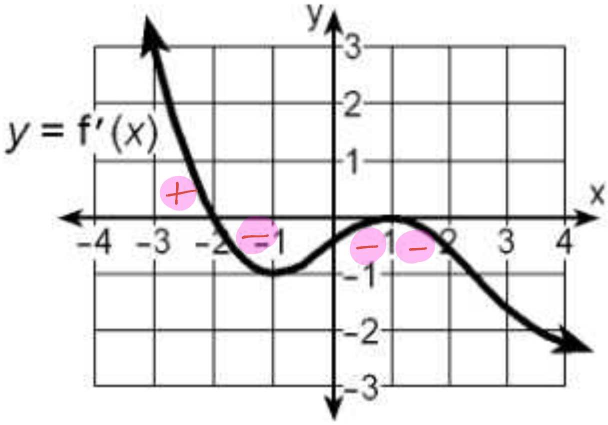
$D_f: \mathbb{R}$

(A) π (B) $\dfrac{2\pi}{3}$ (C) $\dfrac{5\pi}{6}$ (D) 0 (E) $\dfrac{\pi}{3}$

$f'(x) = \frac{1}{2} + \cos x = 0$
$\cos x = -\frac{1}{2}$
$x = \frac{2\pi}{3}$

π 0

A ______ 5. The graph of the derivative, $f'(x)$, of a function $f(x)$ is shown below



At what value of x does $f(x)$ have a local maximum?

(A) -2 (B) -1 (C) 3 (D) 1 (E) 0

$f'=0$

cvs: $x=-2, 1$

f has a local max at $x=-2$, since f' changes from pos to neg at $x=-2$.

D ______ 6. Selected values for the derivative, $f'(x)$, of a differentiable function $f(x)$ are shown in the table below.

x	1	2	3	4	5	6
$f'(x)$	8	4	0	-4	-8	-12

$\to f''<0\ \forall x \to f$ is concave down $\forall x$

If $\boxed{f'(x) \text{ is strictly decreasing,}}$ which of the following statements **must** be true?

~~(A)~~ The graph of $f(x)$ is symmetric with respect to the line $x=3$ (Not necessarily, missing y-values)

~~(B)~~ $f(x)$ is concave up for for all x, cc down, not cc up

~~(C)~~ $f(x)$ changes concavity at $x=3$, Not necessarily, no info immediately on either side of $x=3$

(D) $f(x)$ has a relative maximum at $x=3$

~~(E)~~ $f(x)$ has a relative minimum at $x=3$

since $f'(3)=0$, $x=3$ is a critical value of $f(x)$.

since $f''(3)<0$ ($\forall x$), by 2nd Deriv Test, f has a local/rel max at $x=3$.

C ______ 7. The function g is defined by the equation $g(x)=6x^5-10x^3$. On what open intervals is the graph of $g(x)$ **concave up**? HINT: $\dfrac{\sqrt{2}}{2} \approx 0.707$

$g'(x)=30x^4-30x^2$

$g''(x)=120x^3-60x$

$g''(x)=60x(2x^2-1)$

$g''(x)=0$

$x=0,\ x=\pm\sqrt{\tfrac{1}{2}}=\pm\tfrac{1}{\sqrt{2}}=\pm\tfrac{\sqrt{2}}{2}\approx\pm 0.707$

(A) $\left(-\infty,-\dfrac{\sqrt{2}}{2}\right)\cup\left(0,\dfrac{\sqrt{2}}{2}\right)$ (B) $\left(-\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2}\right)$ (C) $\left(-\dfrac{\sqrt{2}}{2},0\right)\cup\left(\dfrac{\sqrt{2}}{2},\infty\right)$

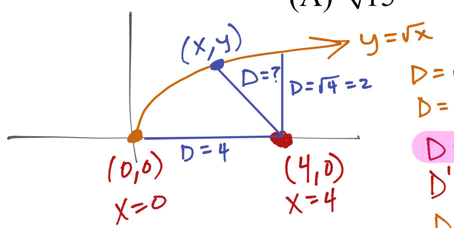
(D) $\left(-\dfrac{\sqrt{2}}{2},\infty\right)$ (D) $\left(-\infty,\dfrac{\sqrt{2}}{2}\right)$

x	-1	$-\sqrt{2}/2$	$-1/2$	0	$1/2$	$\sqrt{2}/2$	1
g''	−		+		−		+

g is cc up on $\left(-\frac{\sqrt{2}}{2},0\right)\cup\left(\frac{\sqrt{2}}{2},\infty\right)$

C 8. The shortest distance from the curve $y=\sqrt{x}$ and the point $(4,0)$ is

(A) $\sqrt{15}$ (B) $\frac{\sqrt{14}}{2}$ (C) $\frac{\sqrt{15}}{2}$ (D) $\frac{7}{2}$ (E) $\sqrt{14}$



$D=\sqrt{(x-4)^2+(y-0)^2}$ constraint $y=\sqrt{x}$

$D=\sqrt{(x-4)^2+y^2}$

$D=((x-4)^2+x)^{1/2}$, $D\in[0,4]$

$D'=\frac{1}{2}((x-4)^2+x)^{-1/2}\cdot[2(x-4)(1)+1]$

$D'=\frac{2x-8+1}{2\sqrt{(x-4)^2+x}}$

$D'=\frac{2x-7}{2\sqrt{(x-4)^2+x}}=0$

$2x-7=0$

$x=\frac{7}{2}=3.5$

By EVT

endpt: $D(0)=4$

endpt: $D(4)=2$

c.v.: $D(3.5)=\sqrt{(3.5-4)^2+3.5}$

$=\sqrt{\frac{1}{4}+\frac{7}{2}}$

$=\sqrt{\frac{15}{4}}$

$=\frac{\sqrt{15}}{2}\approx\frac{3.\ldots}{2}\approx 1.\ldots$

$1.\ldots<2<4$

So, the minimum distance is $\frac{\sqrt{15}}{2}$.

C 9. The diagram below shows a rectangle inscribed in a semicircle.

$A=2xy$ constraint $x^2+y^2=2^2$

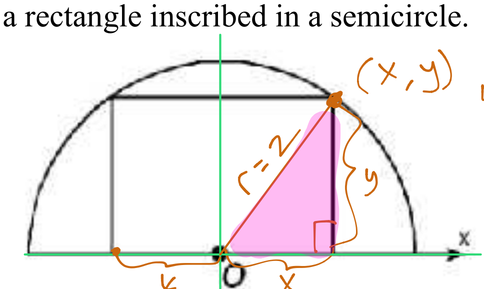
$y=\sqrt{4-x^2}$

$A=2x(4-x^2)^{1/2}$, $x\in[-2,2]$

$A'=(2)(4-x^2)^{1/2}+(2x)(\frac{1}{2})(4-x^2)^{-1/2}(-2x)$

$A'=\frac{2\sqrt{4-x^2}}{1}\left(\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}\right)-\frac{2x^2}{\sqrt{4-x^2}}$

$A'=\frac{2(4-x^2)-2x^2}{\sqrt{4-x^2}}=0$



$A'=0$

when $2(4-x^2)-2x^2=0$

$8-2x^2-2x^2=0$

$8-4x^2=0$

$4x^2=8$

$x^2=2$

$x=\pm\sqrt{2}$

If the radius of the semicircle is 2 meters, what is the maximum area, in square meters, of the rectangle?

(A) $4\sqrt{2}$ (B) $2\sqrt{2}$ (C) 4 (D) 8 (E) 2

So, max area when $x=\sqrt{2}$

$A=2(\sqrt{2})\sqrt{4-(\sqrt{2})^2}$

$=2\sqrt{2}\sqrt{4-2}$

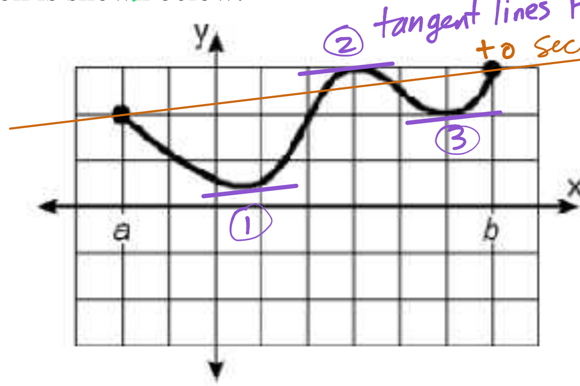
$=2\sqrt{2}\sqrt{2}$

$=2\cdot 2$

$=4$

D 10. The graph of a function is shown below.

tangent lines parallel to secant line



On the closed interval $[a,b]$, at how many points is the Mean Value Theorem satisfied?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Part II: Free Response

11. Let f be the function defined by $f(x) = 36x^{1/3} - 9x^{4/3}$

¡cusp!ert! at $x=0$

(a) What is the domain of $f(x)$?

$f(x) = 36\sqrt[3]{x} - 9\sqrt[3]{x^4}$

$D_f: \mathbb{R}$ ($\checkmark_1$) or $\{x \mid x \in \mathbb{R}\}$ or $x \in (-\infty, \infty)$

(b) Show that $f'(x) = \dfrac{-12(x-1)}{\sqrt[3]{x^2}}$. Show the work that leads to your answer.

$f(x) = 36x^{1/3} - 9x^{4/3}$

$f'(x) = \frac{36}{3}x^{-2/3} - \frac{36}{3}x^{1/3}$ ($\checkmark_2$) or

$f'(x) = 12x^{-2/3} - 12x^{1/3}$

Method 1: Factor out least powers

$f'(x) = 12x^{-2/3}[1 - x]$

$= \dfrac{12(1-x)}{\sqrt[3]{x^2}}$

$= \dfrac{-12(x-1)}{\sqrt[3]{x^2}}$ ($\checkmark_3$) must show ALL steps

OR

Method 2: common denominator

$f'(x) = \dfrac{12}{x^{2/3}} - \dfrac{12x^{1/3}}{1}\left(\dfrac{x^{2/3}}{x^{2/3}}\right)$

$f'(x) = \dfrac{12 - 12x}{x^{2/3}}$

$f'(x) = \dfrac{-12(x-1)}{\sqrt[3]{x^2}}$

(c) Find the intervals on which f is decreasing.

$f'(x) = DNE$ when $\sqrt[3]{x^2} = 0$
c.v.: $x = 0$ ($\checkmark_4$)
(cusp/vert tangent)

$f'(x) = 0$ when $-12(x-1) = 0$
c.v.: $x = 1$ ($\checkmark_5$)
(Horiz Tangent)

No discons for f' chart.
Only 2 cvs.

x	-1	0	$1/2$	1	2
$f'(x)$	$+$		$+$		$-$

$f'(x) = \frac{-12(x-1)}{\sqrt[3]{x^2}}$

So, $f(x)$ is decreasing on $(1, \infty)$ ($\checkmark_6$) or $[1, \infty)$

(d) At each critical value, determine if $f(x)$ has a local maximum, a local minimum, or neither. Justify.

f has neither a local max nor local min at $x=0$, ($\checkmark_7$)
since f' is positive on either side of $x=0$. ($\checkmark_8$)

f has a local max at $x=1$, ($\checkmark_9$)
since f' changes from positive to negative at $x=1$. ($\checkmark_{10}$)

10 F.R. points