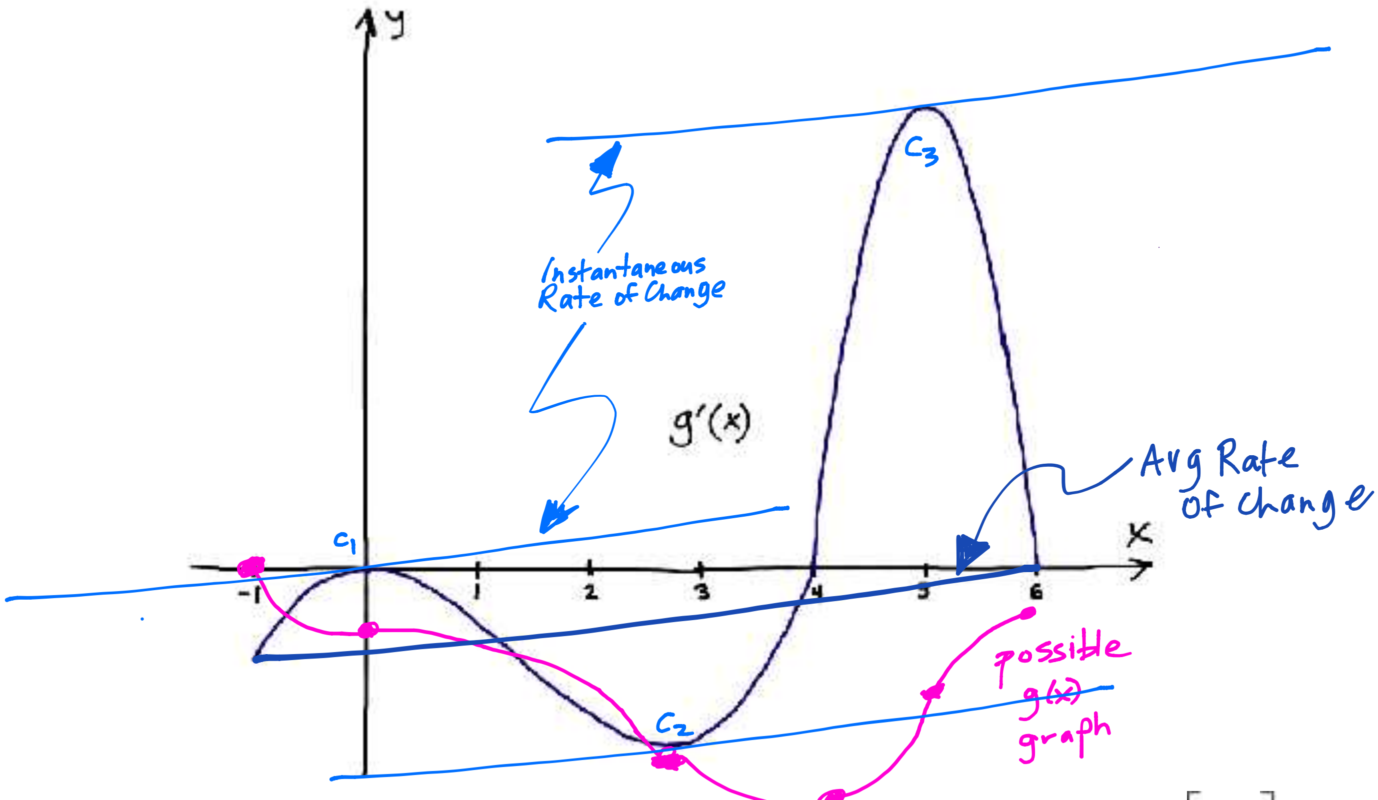
Name KEY Date Friday, 1/22/2016 Period 19 points TOTAL

AB Calculus Test: 3.1-3.5 No Calculator

Part I: Multiple Choice



Use the graph above for questions 1 – 4. Let g be a function that is differentiable on the interval $[-1,6]$. The graph of the continuous function g', the derivative of g, is given above.

C 1. At what value of x can the absolute minimum of g occur?

(A) -1 (B) 3 (C) 4 (D) 5 (E) 6

B 2. How many local extrema does the graph of g have on the interval $[-1,6]$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

D 3. How many inflection values does the graph of g have on the interval $[-1,6]$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

D 4. How many values of x satisfy the Mean Value Theorem for the function $g'(x)$ on the interval $[-1,6]$?

(Blue lines on graph)

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

B 5. If $f(x)$ is a differentiable function such that $f(11)=19$ and $f'(x)\le 2$ for all x, what is the smallest possible value of $f(-1)$?

(A) 43 (B) -5 (C) 9 (D) -29 (E) 17

by MVT, $f'(x) = \dfrac{f(11)-f(-1)}{11-(-1)} \le 2$ } $\dfrac{19-f(-1)}{12} \le 2$ } $19 - f(-1) \le 24$, $-f(-1) \le 5$, $f(-1) \ge -5$

B 6. Use the EVT to find the range of the function $f(x)=\frac{4}{x}+2x^2$ on the interval $\frac{1}{2}\le x\le\frac{3}{2}$.

(A) $6\le f(x)\le\frac{43}{6}$ (B) $6\le f(x)\le\frac{17}{2}$ (C) $\frac{5}{2}\le f(x)\le\frac{21}{2}$ (D) $6\le f(x)\le\frac{21}{2}$ (E) $\frac{5}{2}\le f(x)\le\frac{43}{6}$

$f'(x)=-\frac{4}{x^2}+4x$

$f'(x)=\frac{-4+4x^3}{x^2}$

$f'(x)=\frac{4(x^3-1)}{x^2}$

$f'(x)=0$ at $x=1$ ← CV on interval

$f(\frac{1}{2})=8+\frac{1}{2}=\frac{17}{2}=8\frac{1}{2}$ ← MAX

$f(1)=4+2=6$ ← MIN

$f(\frac{3}{2})=\frac{8}{3}+\frac{9}{2}=\frac{16+27}{6}=\frac{43}{6}=7\frac{1}{6}$

So Range is $y\in[6,\frac{17}{2}]$

A 7. If $f'(x)=\left[x(x+5)^4(3x-1)^{-3/5}\right]^3$ for some continuous function f, then f has which of the following?

$f'=0$ at $x=0,-5,\frac{1}{3}$ → c.v.s

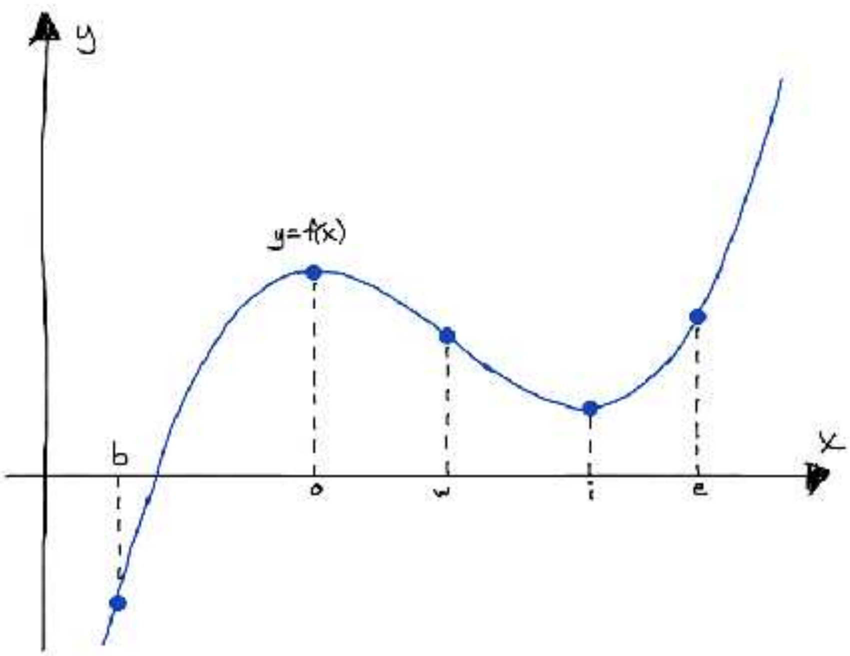
x	-6	-5	-1	0	$\frac{1}{4}$	$\frac{1}{3}$	1
f'	$+$		$+$		$-$		$+$

I. Local maximum at $x=0$ ✓

II. Local maximum at $x=-\frac{1}{3}$

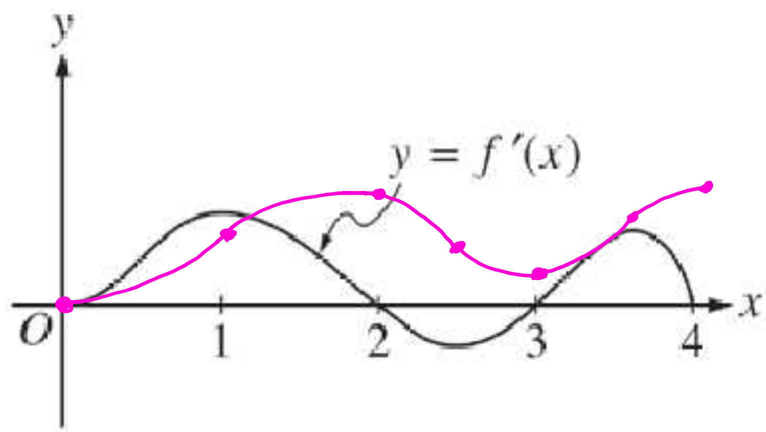
III. Local minimum at $x=-5$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III



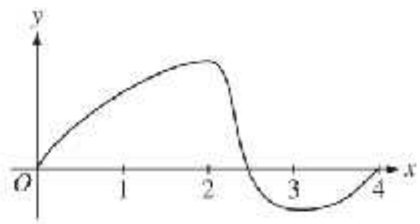
B 8. The function f is shown above with marked locations, b, o, w, i, and e. Of the following, which has the smallest value?

(A) $f'(b)$	(B) $f''(o)$	(C) $f''(w)$	(D) $f'(i)$	(E) $f''(e)$
>0	<0	$=0$	$=0$	>0
(pos slope)	(cc dwn)	(inflection pt.)	(critical value)	(cc up)
$+$	$-$	0	0	$+$

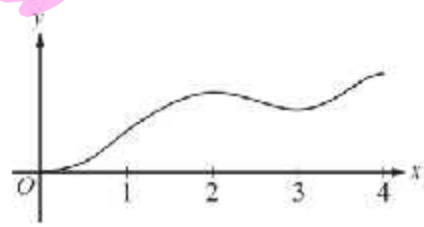


B
______ 9. The figure above shows the graph of f', the derivative of the function f. If $f(0)=0$, which of the following could be the graph of f?

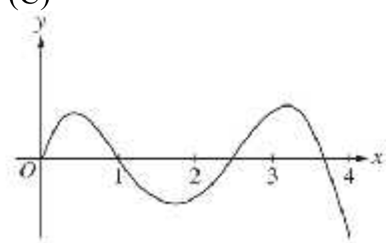
(A)



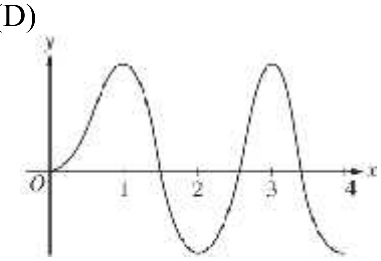
(B)



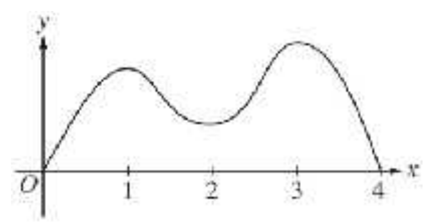
(C)



(D)



(E)



D
______ 10. Selected information is given below about a continuous function $f(x)$ that is continuous for all real numbers.

	$x<0$	$x=0$ C.V.	$0<x<2$	$x=2$ C.V.
$f(x)$	negative	0	positive	4
$f'(x)$	positive	DNE	negative	0
$f''(x)$	negative	DNE	negative	-3

Which of the following must be true about the function $f(x)$

✓ I. $f(x)$ has a local maximum of 4 , $f''(2)=-3<0$ Local Max of 4 at $x=2$ by 2nd Deriv Test.

✓ II. $f(x)$ has a local maximum at 0 f' changes from pos to neg at $x=0$ & $f'(0)=$ DNE while $f(0)$ is defined so f has a local max at $x=0$ by 1st Deriv Test

III. $f(x)$ has an inflection value at 2

(A) I only (B) II only (C) II and III only (D) I and II only (E) I, II, and III

No, $f''(2)=-3\neq 0$

① C
② B
③ D
④ D
⑤ B
⑥ B
⑦ A
⑧ B
⑨ B
⑩ D

Part II: AB Free Response:

Suppose f is a function given by $f(x) = -x^{1/3} - x^{2/3}$,

(a) Show that $f'(x) = \dfrac{1+2x^{1/3}}{-3x^{2/3}}$.

$f(x) = -x^{1/3} - x^{2/3}$

$f'(x) = -\frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-1/3}$ ✓1

$f'(x) = -\frac{1}{3}x^{-2/3}\left[1+2x^{1/3}\right]$

$\boxed{f'(x) = \dfrac{1+2x^{1/3}}{-3x^{2/3}}}$ ✓2

$= \dfrac{1+2\sqrt[3]{x}}{-3(\sqrt[3]{x})^2}$

(b) Determine the x-coordinates of any local extrema of $f(x)$? Justify your answer using the 1st Derivative Test.

$f'(x) = \text{DNE}$ when $-3x^{2/3} = 0$

$\boxed{x=0}$ ✓3

$f'(x) = 0$ when $1+2x^{1/3} = 0$

$x^{1/3} = -\frac{1}{2}$

$\boxed{x = -\frac{1}{8}}$ ✓4

x	-1	$-1/8$	$-1/27$	0	1
f'	$+$		$-$		$-$

✓7 for chart

f has a local max at $x = -1/8$ ✓5
since f' changes from pos to neg at $x = -\frac{1}{8}$. ✓6

(c) Determine the intervals on which $f(x)$ is concave up.

$f'(x) = -\frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-1/3}$

✓8 $f''(x) = \frac{2}{9}x^{-5/3} + \frac{2}{9}x^{-4/3}$

$f''(x) = \frac{2}{9}x^{-5/3}\left[1 + x^{1/3}\right]$

$f''(x) = \dfrac{2(1+x^{1/3})}{9x^{5/3}}$

$f''(x) = \dfrac{2(1+\sqrt[3]{x})}{9(\sqrt[3]{x})^5}$

$f'' = \text{DNE}$: $\boxed{x=0}$ p.i.v.

$f'' = 0$: $2(1+\sqrt[3]{x}) = 0$

$\sqrt[3]{x} = -1$

$\boxed{x=-1}$ p.i.v.

x	-8	-1	$-1/8$	0	1
f''	$+$		$-$		$+$

$\boxed{f \text{ is concave up on } (-\infty,-1) \cup (0,\infty)}$ ✓9

*8th check can also be earned using the Quotient Rule.

$f'(x) = \dfrac{1+2x^{1/3}}{-3x^{2/3}}$

$f'' = \dfrac{(-3x^{2/3})(\frac{2}{3}x^{-2/3}) - (1+2x^{1/3})(-2x^{-1/3})}{9x^{4/3}}$ ✓8