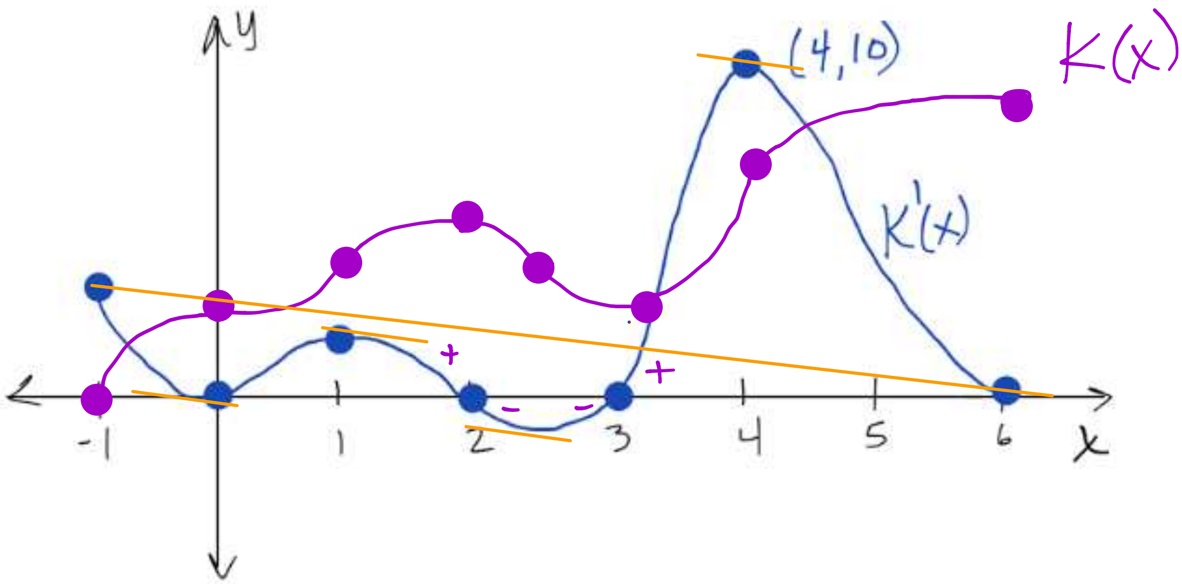
Name KEY Date 11/14/2017 Seasonal Spice Pumpkin

AB Calculus Test: 3.1-3.5 No Calculator 19 pt Test

Part I: Multiple Choice—Put the CAPITAL letter in the space to the left of each question number.



Use the graph above for questions 1 – 4. Let k be a function that is differentiable on the interval $[-1,6]$. The graph of the continuous function $k'(x)$, the derivative of k, is given above. The graph of $k'(x)$ has x-intercepts at $x = 0$, $x = 2$, $x = 3$, and $x = 6$.

E 1. At what value of x can the absolute maximum of k occur?

(A) -1 (B) 3 (C) 4 (D) 5 (E) 6

C 2. How many local extrema does the graph of k have on the interval $[-1,6]$? Local Max @ x=2 Local Min @ x=3

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

E 3. How many inflection values does the graph of k have on the interval $[-1,6]$? Local Extrema of k'(x)

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

E 4. How many values of x satisfy the Mean Value Theorem for the function $k'(x)$ on the interval $[-1,6]$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

D 5. If $f(x)$ is a differentiable function such that $f(10) = 29$ and $f'(x) \le 3$ for all x, what is the smallest possible value of $f(-1)$?

(A) 62 (B) 33 (C) 3 (D) -4 (E) -33

$$f'(x) = \frac{f(10) - f(-1)}{10-(-1)} \le 3 \quad \frac{29 - f(-1)}{11} \le 3 \quad 29 - f(-1) \le 33 \quad -f(-1) \le 4 \quad f(-1) \ge -4$$

A 6. Use the EVT to find the range of the function $f(x) = 2x^3 + 3x^2 - 12x - 1$ on the interval $-1 \le x \le 2$.

(A) $y \in [-8, 12]$ (B) $y \in [-8, 3]$ (C) $y \in [3, 12]$ (D) $y \in [-8, 19]$ (E) $y \in [3, 19]$

$f'(x) = 6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$\underbrace{x = -2}_{\text{Not in Interval}}$, $x = 1$

$f(-1) = -2 + 3 + 12 - 1 = 12$

$f(1) = 2 + 3 - 12 - 1 = -8$

$f(2) = 16 + 12 - 24 - 1 = 3$

Range: $[-8, 12]$

MIN MAX

C 7. If $M'(x) = x^2(x-4)^3(2x+1)^{-4/3}$ for some continuous function M, then M has which of the following?

$\dfrac{x^2(x-4)^3}{\sqrt[3]{(2x+1)^4}}$

$M' = 0$

$x = 0, x = 4, x = -\frac{1}{2}$

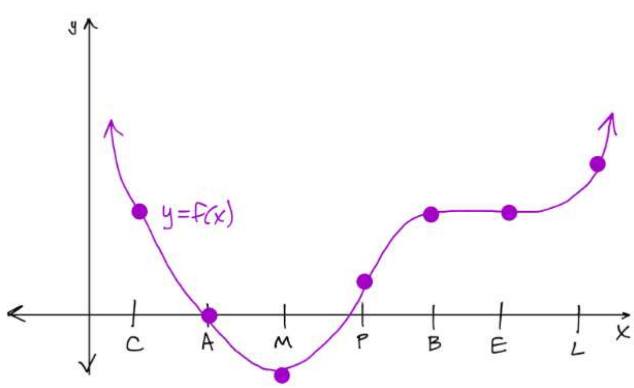
	$-\frac{1}{2}$		0		4	
x	-1		$-\frac{1}{4}$	1		5
M'	$-$		$-$	$-$		$+$

I. Local minimum at $x = 0$

II. Local maximum at $x = -\frac{1}{2}$

III. Local minimum at $x = 4$

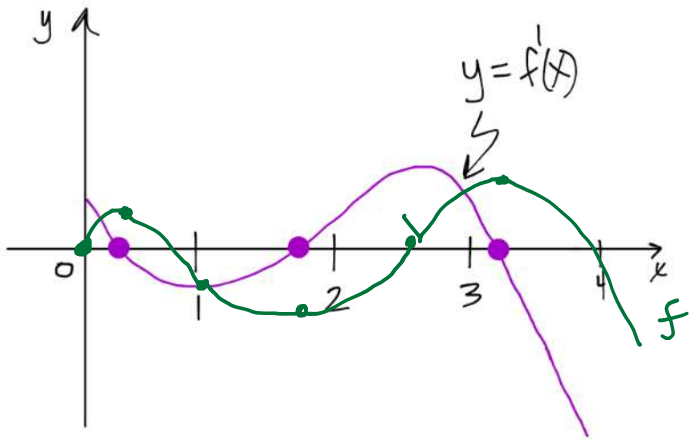
(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III



D 8. The function f is shown above with dots corresponding to the marked locations, C, A, M, P, B, E, and L. Of the following, which has the LARGEST value?

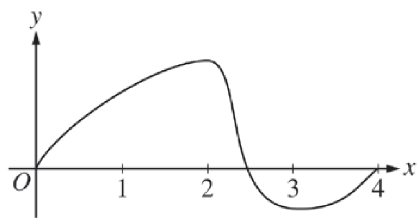
(A) $f'(C)$ (B) $f''(B)$ (C) $f'(M)$ (D) $f''(L)$ (E) $f(A)$

Neg Neg 0 Pos 0

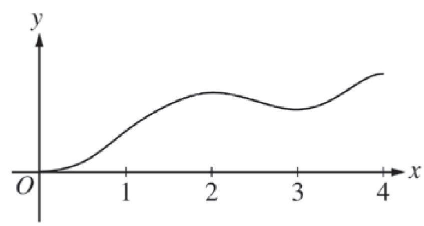


C 9. The figure above shows the graph of f', the derivative of the function f. If $f(0)=0$, which of the following could be the graph of f?

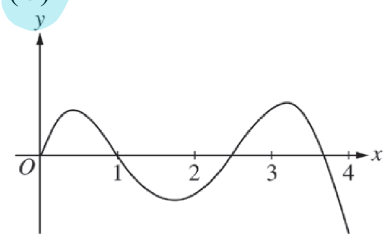
(A)



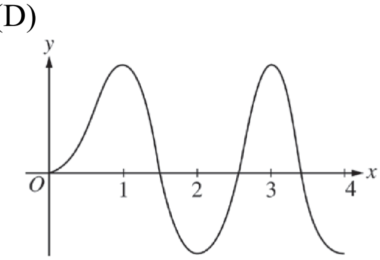
(B)



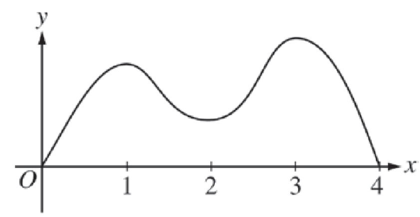
(C)



(D)



(E)



D 10. Selected information is given below about a continuous function $f(x)$ that is continuous for all real numbers.

	$x<-2$	$x=-2$	$-2<x<1$	$x=1$
$f(x)$	positive	6	negative	3
$f'(x)$	positive	DNE	negative	0
$f''(x)$	positive	DNE	negative	2

Which of the following must be true about the function $f(x)$

I. $f(x)$ has a local minimum of 3 ← Local Min of 3, $f''(1)>0$ ✓

II. $f(x)$ has a local maximum at -2 $f'(-2)=0$, f' from + to −

III. $f(x)$ has an inflection value at 1 ← No, $f''(1)=2\neq 0$

(A) I only (B) II only (C) II and III only (D) I and II only (E) I, II, and III

Part II: AB Free Response—Show all work in the space provided. Use proper notation (always).

Suppose f is a function given by $f\left(x\right) = 6x^{2/3} - 3x^{4/3}$,

(a) Show that $f'\left(x\right) = \dfrac{4\left(1-\sqrt[3]{x^2}\right)}{\sqrt[3]{x}}$.

$f'(x) = 4x^{-1/3} - 4x^{1/3}$ ✓1

$f'(x) = 4x^{-1/3}\left(1 - x^{2/3}\right)$

$f'(x) = \dfrac{4\left(1-\sqrt[3]{x^2}\right)}{\sqrt[3]{x}}$ ✓2

(b) Determine the x-coordinates of any local max/mins of $f\left(x\right)$? Justify your answer using the 1st Derivative Test.

$D_f : \mathbb{R}$

$f'(x) = \dfrac{4\left(1-\sqrt[3]{x^2}\right)}{\sqrt[3]{x}}$

$f' = DNE$ $\quad$ $f' = 0$

$x = 0$ $\quad$ $1 - \sqrt[3]{x^2} = 0$

$x = -1, 1$

	-1	0	1	
x	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
f'	$+$	$-$	$+$	$-$

✓3 ✓4

$\ast$ f has local maximums at $x = -1$ & $x = 1$ ✓5

since f' changes from pos to neg at $x = \pm 1$.

$\ast$ f has a local min at $x = 0$ ✓6 ✓7

since f' changes from neg to pos at $x = 0$

(c) Determine the intervals on which $f\left(x\right)$ is concave down. Justify.

$f'(x) = \dfrac{4\left(1-\sqrt[3]{x^2}\right)}{\sqrt[3]{x}}$

from (a): $f'(x) = 4x^{-1/3} - 4x^{1/3}$

$f''(x) = -\frac{4}{3}x^{-4/3} - \frac{4}{3}x^{-2/3}$ ✓8

$f''(x) = -\frac{4}{3}x^{-4/3}\left(1 + x^{2/3}\right)$

$f''(x) = \dfrac{-4\left(1+\sqrt[3]{x^2}\right)}{3\sqrt[3]{x^4}} < 0$ for all $x \neq 0$

So, f is cc down $\forall x \neq 0$ ✓9