Name KEY Date Period

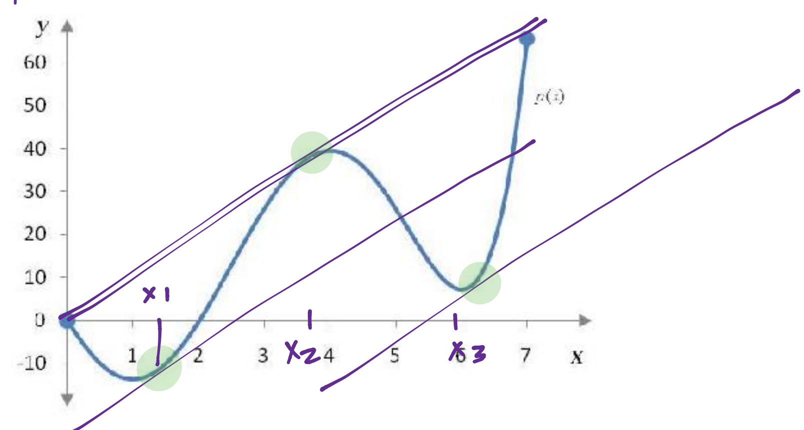
TEST: 3.1-3.5, NO CALCULATOR

Part I: Multiple Choice: Put the letter in the letter place. Be sure it's write, wright, rite, . . . correct.

B 1. If $f(x)=3x^5-4x^4+7x^3-e^x$, what is $\lim_{h\to 0}\frac{f^{(5)}(0+h)-f^{(5)}(0)}{h}$?

(A) 1 (B) -1 (C) 359 (D) 361 (E) 0

$f^{(6)}(0) = -e^0 = -1$



D 2. The graph of a differentiable function $p(x)$ is shown above. For how many values of x on $[0,7]$ is the MVT satisfied?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

E 3. The function g is differentiable & cont. and non-linear for $-4\le x\le 6$. If $g(-4)=-\frac{1}{2}$ and $g(6)=\frac{1}{2}$, then for some $r\in(-4,6)$, which of the following must be true?

I. $g(r)=0$ IVT ✓ II. $g'(r)=0$ ✓ MVT III. $g'(r)=\frac{1}{10}$

$\frac{\frac{1}{2}-(-\frac{1}{2})}{6-(-4)} = f'$ $f'=\frac{1}{10}$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

B 4. Let $f(x)$ be a differentiable function such that $f(-b)=3$, $b>0$, and $f'(x)\le 5$ for all x. What is the largest possible value of $f(b)$? $f'(x)=\frac{f(b)-f(-b)}{b-(-b)}\le 5$

(A) $10b$ (B) $3+10b$ (C) $5b$ (D) $3+5b$ (E) $20b$

so $\frac{f(b)-3}{2b}\le 5$ $f(b)\le 10b+3$

D 5. The graph of a twice-differentiabl function h is shown at right. Arrange the following expressions from smallest to largest.

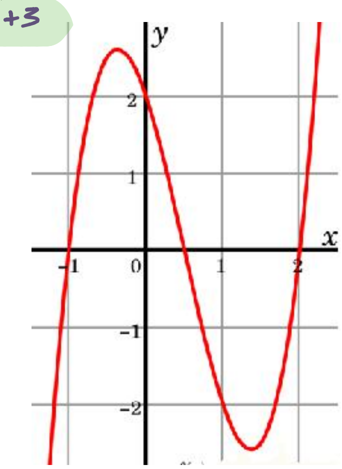
I. $h(-1)+h(2) = 0+0=0$

II. $h'(-1)+h'(2) =$ pos + pos = pos

III. $h''(-1)-h''(2) =$ neg − pos = neg

(A) III, II, I (B) II, III, I (C) I, III, II (D) III, I, II (E) I, II, III

III, I, II



B 6. If $f'(x)=\left[x(x-5)^3(2x-3)^{-2/3}\right]^3$ for some continuous function f, then f has which of the following?

preserves sign

CVS: $x=0, 5, \frac{3}{2}$

I. Local minimum at $x=0$

II. Local minimum at $x=5$

III. Local maximum at $x=\frac{3}{2}$

x: -1, 0, 1, $3/2$, 2, 5, 6
f': $+$ | $-$ | $-$ | $+$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

D 7. It was reported this week that the price of gasoline is still falling, but not as fast as it was last week. If P is current price of gasoline, which of the following statements is true?

I. $P>0$ II. $P<0$ III. $\frac{dP}{dt}>0$ IV. $\frac{dP}{dt}<0$ V. $\frac{d^2P}{dt^2}>0$ VI. $\frac{d^2P}{dt^2}<0$

pos, dec, ccu
$P>0$, $P'<0$, $P''>$

(A) I, III, V only (B) I, IV, VI only (C) I, III, VI only (D) I, IV, V only (E) II, III, VI only

D 8. If $f'(x)=-e^{2x}\left(5+2x-x^2\right)$, for what values of x is f concave down? $f''<0$

deriv already

$f''=-2e^{2x}(5+2x-x^2)-e^{2x}(2-2x)=0$

$2e^{2x}[-5-2x+x^2-1+x]=0$, $2e^{2x}(x-3)(x+2)=0$

$2e^{2x}(x^2-x-6)=0$, $x=3,-2$

x: -3, -2, 0, 3, 4
f'': $+$ | $-$ | $+$

(A) $(-\infty,-3)\cup(2,\infty)$ (B) $(-\infty,-2)\cup(3,\infty)$ (C) $(-3,2)$ (D) $(-2,3)$ (E) $\left(-\sqrt{7},\sqrt{7}\right)$

C 9. A critter is moving along a horizontal wire with position function $x(t)=t^4-8t^3+18t^2-216t+1$ for $t\in[0,4]$. What is the critter's velocity at the time when the critter attains its minimum acceleration?

(A) -216 (B) -240 (C) -208 (D) -407 (E) -12

minimize $x''(t)=a(t)$

find cvs of $a(t)$: $a'(t)=$

$x'=4t^3-24t^2+36t-216=v(t)$

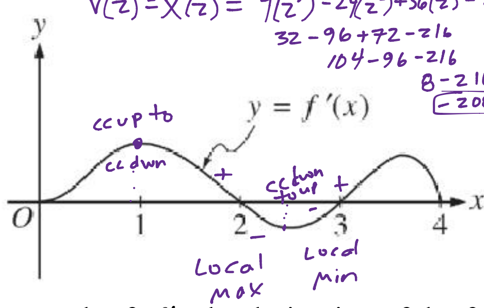
$x''=a(t)=12t^2-48t+36$

$a'(t)=24t-48=0$

$\boxed{t=2}$ abs min of location of $a(t)$

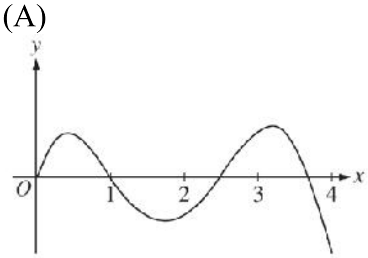
$a'(t)$: $-$ | $+$

$V(2)=x'(2)=4(2^3)-24(2^2)+36(2)-216$
$32-96+72-216$
$104-96-216$
$8-216$
$\boxed{-208}$

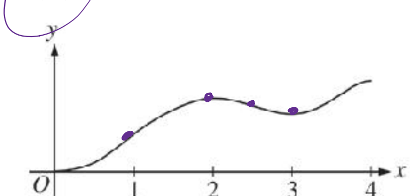


B 10. The figure above shows the graph of f', the derivative of the functin f. If $f(0)=0$, which of the following could be the graph of f?

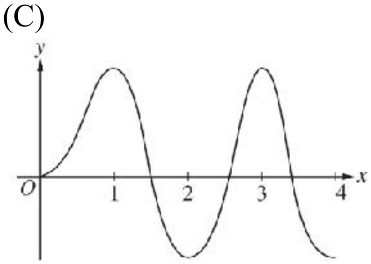
(A)



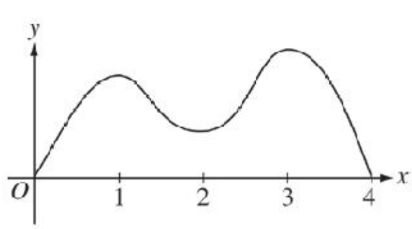
(B)



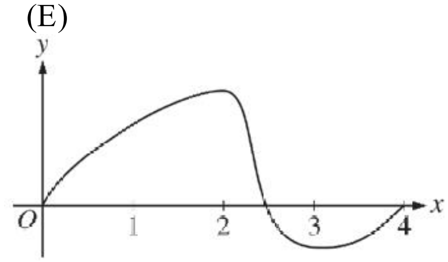
(C)



(D)

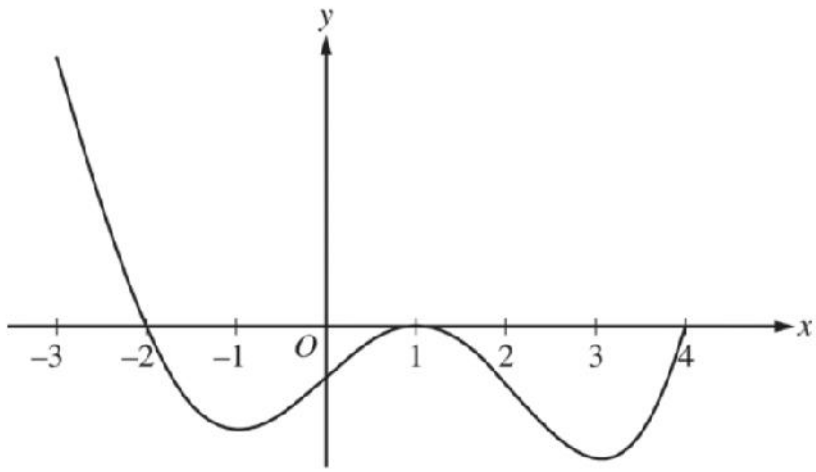


(E)



Part II: Free Response 2015 #5

Say what you want, but be sure to document and say it correctly with correct documentation.



Graph of f'

11. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3,4]$. The graph of f' has horizontal tangents at $x=-1$, $x=1$, and $x=3$.

(a) Find all x-coordiantes at which f has a relative maximum. Give a reason for your answer.

$f'=0$

$x=-2, 1, 4$

f has a local max at $x=-2$ ✓1 ← IDs $x=-2$ as candidal

since f' changes from pos to neg at $x=-2$. ✓2

(b) On what open itervals contained in $-3<x<4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

$(-2,-1) \cup (1,3)$ ✓3

since $f'<0$ and the slopes of $f'<0$ on these intervals ✓4

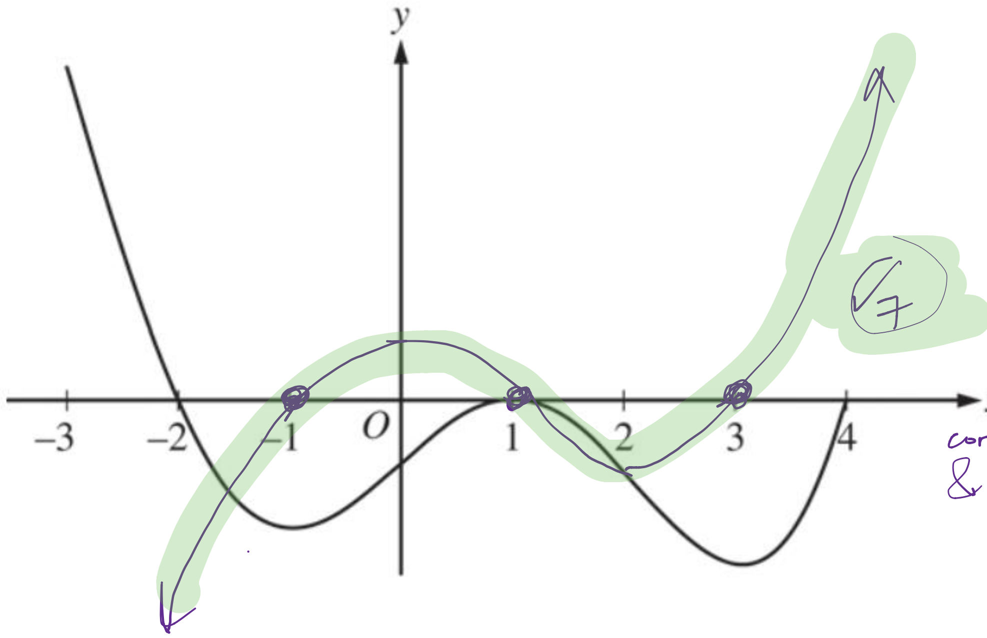
or

since $f'<0$ and dec on these intervals.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

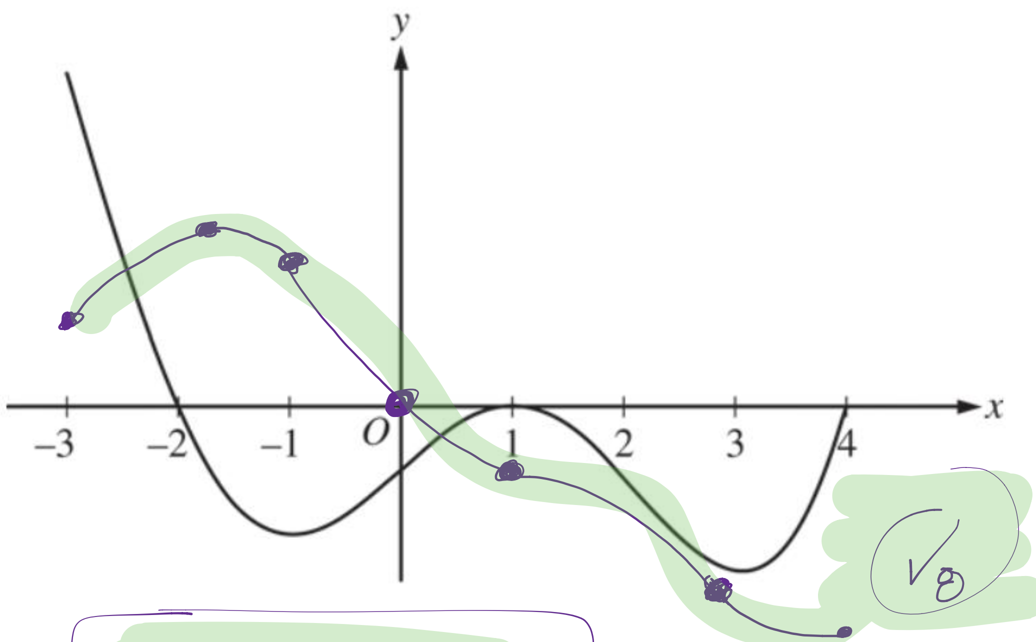
f has inflection values at $x=-1, 1, 3$ since ✓5 the slopes of f' change from neg to pos at $x=-1$ & $x=3$ and from pos to neg at $x = x=1$. ✓6

(d) On the graph below, on the same axes as the graph of f' sketch a possible graph of $f''(x)$



correct endbehavior
& x-ints.

(e) If $f(0) = 0$, on the graph below, on the same axes as the graph of f' sketch a possible graph of $f(x)$.
For $-3 \le x < 4$, at what value of x does f attain it global maximum value?



*local extrema
*and curvature
*passes through origin

f attains its global max at x = -2 √9