Name KEY Date Evergreen Tree

TEST: 3.1-3.4, NO CALCULATOR

Part I: Multiple Choice—Put the CAPITAL letter of the correct answer in the space to the left of the question number.

A 1. Find the local minimum value or the local maximum value of $y = x^3 - 6x^2 + 9x + 1$.

(A) local max of 5 (B) local max of 1 (C) local max of 3 (D) local min of 5 (E) local min of -3

$y(1) = 1 - 6 + 9 + 1$
$y(1) = 5$ (local max)

$y(3) = 27 - 6(9) + 27 + 1$
$y(3) = 1$ (local min)

$y' = 3x^2 - 12x + 9 = 0$
$3(x^2 - 4x + 3) = 0$
$3(x-3)(x-1) = 0$
CVs: $x = 1, x = 3$

x	-2	1	2	3	4
y'	$+$		$-$		$+$
		Local max at $x=1$		Local min at $x=3$	

D 2. The function $f(x) = x^3 - 3x^2 + 3$ has an inflection point at

(A) $(0,3)$ (B) $(1,3)$ (C) $(2,-1)$ (D) $(1,1)$ (E) Does not have one

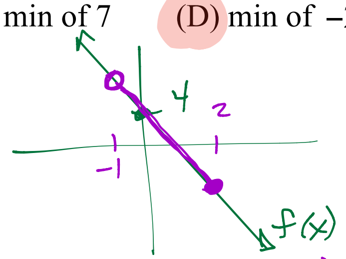
$f'(x) = 3x^2 - 6x$
$f''(x) = 6x - 6 = 0$
$6(x-1) = 0$
p.I.V.: $x = 1$

x	-1	1	2
f''	$-$		$+$

$f(1) = 1 - 3 + 3$
$f(1) = 1$
So f has an inflection point at $(1,1)$

D 3. Find the absolute minimum value or the absolute maximum value of $f(x) = -3x + 4$ on the interval $(-1, 2]$.

(A) max of 7 (B) max of -2 (C) min of 7 (D) min of -2 (E) neither exist



min of $f(2) = -2$

$f'(x) = -3 \neq 0$
No CVs.
$f(2) = -6 + 4 = -2$

D 4. The f be a function such that it is continuous on $[a,b]$, it is differentiable on (a,b), and $f(a) = f(b)$, then there exists a number c in (a,b) such that $f'(0) = 0$. This theorem is known as:

(A) Extreme Value Theorem (B) Intermediate Value Theorem (C) Mean Value Theorem (D) Rolle's Theorem (E) Fundamental Theorem of Calculus

A 5. How many critical values of the function f defined by $f(x) = \dfrac{x}{x^2-9}$ are there?

(A) none (B) 1 (C) 2 (D) 3 (E) 4

$D_f: \{x \mid x \neq -3, 3\}$

VAs @ $x=-3$, $x=3$

$f'(x) = \dfrac{(x^2-9)(1)-(x)(2x)}{x^2-9}$

$f'(x) = \dfrac{x^2-9-2x^2}{x^2-9}$

$f'(x) = \dfrac{-x^2-9}{x^2-9}$

$f'(x) = \dfrac{-(x^2+9)}{x^2-9}$

No critical values

$f' = DNE$
$x^2-9=0$
$\cancel{x=\pm 3}$
VAs @ $x=\pm 3$

$f'=0$
$-(x^2+9)=0$
$x^2=-9$
No solutions

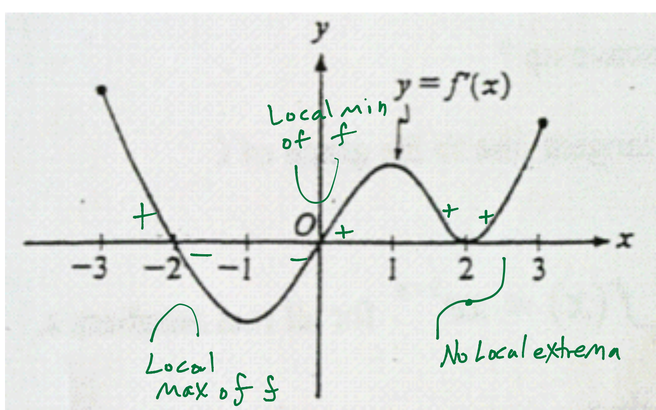
D 6. Let $f''(x) = x(x-3)^2(x+5)$, the second-derivative of a continuous function $f(x)$, on what open interval is the graph of $f(x)$ concave down?

(A) $(-\infty,-5)\cup(-3,\infty)$ (B) $(-\infty,-5)\cup(0,\infty)$ (C) $(0,3)$ (D) $(-5,0)$ (E) $(-\infty,-5)$

$f''=0$
@ $x=0,3,-5$ (pivs)

x	-6	-5	-1	0	1	3	4
f''	+		−		+		+

f is cc down on $(-5,0)$



C 7. The figure above shows the graph of f', the derivative of the function f on the closed interval $-3 \le x \le 3$. If f' has 3 zeros on $-3 < x < 3$, how many relative/local extrema does f have on $-3 < x < 3$?

max or min

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

x	$f(x)$	$f'(x)$	$f''(x)$
-3	5	8	8
-2	3	-12	0
1	2	0	-5
5	1	4	4

A 8. The table above gives select values for a twice-differentiable function $f(x)$. Which of the following must be true regarding $f(x)$?

(A) $f(x)$ has a local maximum at $x = 1$.

(B) $f(x)$ has a local minimum at $x = 1$

(C) $f(x)$ has an inflection value at $x = -2$

(D) $f(x)$ is decreasing on $(-3,1)$

(E) $f(x)$ is concave up on $(-8,0)$

$f'(1) = 0$

$f''(1) < 0$

f is concave down at a critical value

so, f has a local max at $x=1$ by the 2nd Deriv Test!

E 9. Find the value that satisfies the conclusion of the Mean Value Theorem for derivatives for the function $f(x) = 3x^2 - 5x + 1$ on the interval $[2,5]$.

(A) 1 (B) $\frac{13}{6}$ (C) $\frac{11}{6}$ (D) $\frac{23}{6}$ (E) $\frac{7}{2}$

$f'(x) = \frac{f(5) - f(2)}{5-2}$

$6x - 5 = \frac{(3(25) - 25 + 1) - (12 - 10 + 1)}{3}$

$6x - 5 = \frac{51 - 3}{3}$

$6x - 5 = \frac{48}{3}$

$6x - 5 = 16$

$6x = 21$

$x = \frac{21}{6}$

$x = \frac{7}{2}$

1 A
2 D
3 D
4 D
5 A
6 D
7 C
8 A
9 E

Part II: Free Response—Show all work with correct notation in the space provided.

10. Consider a differentiable function $f(x)$ having domain of all positive real numbers, and for which it is known that $f'(x) = (6-x)x^{-2}$ for $\boxed{x > 0}$.

(a) If $f(5) = 2$, write an equation of the tangent line to $f(x)$ at $x = 5$.

$$f'(x) = \frac{6-x}{x^2}$$

$$f'(5) = \frac{(6-5)}{5^2}$$

$$f'(5) = \frac{1}{25}$$ √1

eq: $y = 2 + \frac{1}{25}(x-5)$ √2

(b) Find the x-coordinate of the critical point of $f(x)$. Determine whether the point is a relative maximum, a relative minimum, or neither for the function $f(x)$. Justify your answer.

$$f'(x) = \frac{6-x}{x^2}$$

$f'(x) = \text{DNE}$
$x^2 = 0$
$x = 0$
but $x > 0$

$f'(x) = 0$
$6 - x = 0$
√3 $\boxed{x = 6}$
Only cv of f

x	5	7
f'	+	−

(critical value 6 between 5 and 7)

Local Max of f

* f has a local max at $x = 6$, √4 since f' changes from pos to neg at $x = 6$. √5

(c) Find all intervals on which the graph of $f(x)$ is concave down. Justify your answer.

$$f'(x) = \frac{6-x}{x^2}$$ √6 √7

$$f''(x) = \frac{(x^2)(-1) - (6-x)(2x)}{(x^2)^2}$$

$$f''(x) = \frac{-x^2 - 12x + 2x^2}{x^4}$$

$$f''(x) = \frac{x^2 - 12x}{x^4}$$

$$f''(x) = \frac{x(x-12)}{x^4}$$

$$\boxed{f''(x) = \frac{x-12}{x^3}}$$

$f'' = \text{DNE}$
$x = 0$
but $x > 0$

$f'' = 0$
$x - 12 = 0$
√8 $\boxed{x = 12}$
only piv of f.

x	11	13
f''	−	+

(0 at the left end; 12 between 11 and 13)

So, f is concave down on $(0, 12)$ (since $x > 0$) since $f'' < 0$ on $(0, 12)$. √9