Name________________________________ Date______________________ Period_____________

TEST: 5.1-5.4, NO CALCULATOR

Part I: Multiple Choice: Put the letter in the letter place. Be sure it's write, wright, rite, . . . correct.

_____ 1. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a local minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

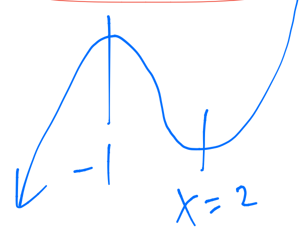
$f' = 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$\boxed{6(x-2)(x+1)} = 0$

$\boxed{x = 2, -1}$

$-2 \quad -1$



__A__ 2. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$ (B) $x < 2$ (C) $0 < x < 4$ (D) $x < 0$ or $x > 4$ only (E) $x > 6$ only

$\boxed{f'' > 0}$

$f'(x) = 3x^2 - 12x$

$f''(x) = \boxed{6x - 12} = 0$

$\boxed{x = 2}$

x	0	2	3
f''	$-$		$+$

__A__ 3. If $f'(x) = (x-2)\cancel{(x-3)^2}(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

~~II. A relative minimum at $x = 3$~~

~~III. A relative maximum at $x = 4$~~

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

$f' = 0, \ x = 2, 3, 4$

x	0	2	2.5	3	3.5	4	5
f'	$+$		$-$		$-$		$+$

C

______ 4. For $x>0$, f is a function such that $f'(x)=\frac{\ln x}{x}$ and $f''(x)=\frac{1-\ln x}{x^2}$. Which of the following is true?

~~(A) f is decreasing for $x>1$~~, and the graph of f is concave down for $x>e$

~~(B) f is decreasing for $x>1$~~, and the graph of f is concave up for $x>e$

(C) f is increasing for $x>1$, and the graph of f is concave down for $x>e$

(D) f is increasing for $x>1$, and the graph of f is concave up for $x>e$

~~(E) f is decreasing for $0<x<1$, and the graph of f is concave down for $0<x<e^{3/2}$~~

$f' = 0$

$\ln x = 0$

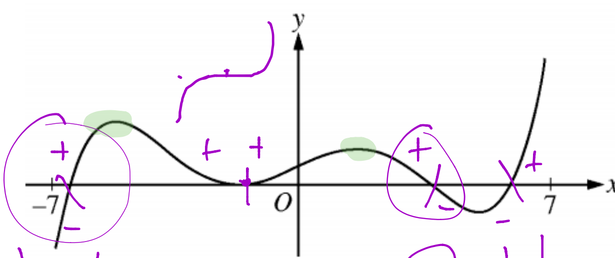
$\boxed{x=1}$

$f''=0$

$1-\ln x = 0$

$\ln x = 1$

$\boxed{x=e}$



Graph of f'

$f'=0$

4 c.v.s

______5. The figure above shows the graph of f', the derivative of the function f on the open interval $-7<x<7$. If f' has four zeros on $-7<x<7$, how many relative maxima does f have on $-7<x<7$?

(A) one (B) two (C) three (D) four (E) five

i.v at $x=1$

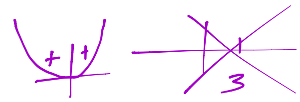
x	0	1	2	3
$f''(x)$	5	0	-7	4

______ 6. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0,2)$ (B) f is decreasing on the interval $(0,2)$

(C) f has a local maximum at $x=1$ (D) The graph of f has a point of inflection at $x=1$

(E) The graph of f changes concavity in the interval $(0,2)$



_____ 7. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)^1(x-6)^1$. What are the x-coordinates of the points of inflection of the graph of f?

$x=0$

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6 only

$f''=0$

$x=0, 3, 6$ p.i.'s

Part II: Free Response
Say what you want, but be sure to document and say it correctly with correct documentation.

2011 AB4 Form B → Cont.   D: $x>0$, $(0,\infty)$

10. Consider a differentiable function f having domain of all positive real numbers, and for which it is known that $f'(x)=(4-x)x^{-3}$ for $x>0$.

    (a) If $f(1)=2$, write an equation of the tangent line to $f(x)$ at $x=1$.

    (b) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

    (c) Find all intervals on which the graph of f is concave down. Justify your answer.   ✓6   ✓7

(a) $(1, 2)$

$f'(1)=(4-1)(1^{-3})$
$=3$ ✓1

eq: $y=y_1+m(x-x_1)$

$y=mx+b$ $\boxed{y=2+3(x-1)}$ ✓2

$y=2+3x-3$

$\boxed{y=3x-1}$

(b) $f'(x)=\boxed{\dfrac{4-x}{x^3}}$

$f'=0$ $f'=DNE$
$4-x=0$ $x^3=0$
✓3 $\boxed{x=4}$ $\cancel{x=0}$

x	3	4	5
f'	+		−

✓4 *** f has a Rel Max at $x=4$
✓5 *** Since f' changes from pos to neg at $x=4$

$f''=(-1)(x^{-3})+(4-x)(-3x^{-4})$

$f''=\dfrac{(x^3)(-1)-(4-x)(3x^2)}{(x^3)^2}$

$f''=\dfrac{-x^3-12x^2+3x^3}{x^6}$

$f''=\dfrac{2x^3-12x^2}{x^6}$

$f''=\dfrac{2x^2(x-6)}{x^6}$

$\boxed{f''=\dfrac{2(x-6)}{x^4}}$

$f''=0$
$x=6$ ✓8

x	5	6	7
f''	−		+

$x>0$

✓9 f is concave down on $(0, 6)$ since $f''<0$ on $(0, 6)$