Name KEY______________________________________ Date__________________ Favorite Coin______

Calculus Test: 2.1 to 3.3. No Calculator

Part I: Multiple Choice—Put the Capital Letter in the blank to the left of each number.

D 1. What is the absolute maximum of the function $f(x)=\frac{1}{2}x^4-\frac{4}{3}x^3+2$ on the interval $[1,3]$?

(A) 2 (B) 3 (C) $\frac{23}{6}$ (D) $\frac{13}{2}$

C 2. Given $f(x)=x^3-3x^2+2x-7$ and $f(g(x))=x=g(f(x))$, what is $g'(f(3))$?

(A) $-\frac{1}{11}$ (B) 11 (C) $\frac{1}{11}$ (D) -11

B 3. If $e^{xy+1}=3$, what is $\frac{dy}{dx}$ at $x=1$?

(A) $\frac{1}{\ln 3}$ (B) $1-\ln 3$ (C) $\ln 3-1$ (D) $\ln 3$

D 4. Given the function $f(x)=x^2+4x-1$, for which of the following values of c on the open interval $(0,5)$ will the conclusion for the Mean Value Theorem be satisfied for the function $f(x)$?

(A) $\frac{9}{2}$ (B) 4 (C) 1 (D) $\frac{5}{2}$

C 5. What are the open interval(s) for which the function $f(x)=\frac{8}{3}x^3-6x^2-36x+7$ is increasing?

(A) $\left(-\frac{3}{2},3\right)$ (B) $(-\infty,3)$ (C) $\left(-\infty,\frac{3}{2}\right)\cup(3,\infty)$ (D) $\left(-\frac{3}{2},\infty\right)$

D 6. $\lim_{h\to 0}\frac{\left[4\cos^4(x+h)+3\sin(x+h)\right]-\left[4\cos^4 x+3\sin x\right]}{h}=$

(A) $16\cos^3 x\sin x+3\cos x$ (B) $-4\cos^3 x\sin x+3\cos x$

(C) $-16\cos^3 x+3\cos x$ (D) $-16\cos^3 x\sin x+3\cos x$

C 7. For $x^2y+\sec y=8$, what is $\frac{dy}{dx}$?

(A) $-2xy\left(x^2\sec y\tan y\right)$ (B) $\frac{x^2y}{\sec y\tan y}$ (C) $\frac{-2xy}{x^2+\sec y\tan y}$ (D) $\frac{-2xy}{x^2-\sec y\tan y}$

A ____ 8. $\frac{d}{dx}\left[\tan^{-1}\left(e^{x^2}\right)\right]=$

(A) $\frac{2xe^{x^2}}{1+e^{2x^2}}$ (B) $\frac{4xe^{x^2}}{1+e^{x^4}}$ (C) $\frac{2x}{1+e^{2x^2}}$ (D) $\frac{2xe^{x^2}}{1+e^{x^2}}$

C ____ 9. If $f(x)=\log\sqrt[4]{(3x+5)^3}$, what is $f'(x)$?

(A) $\frac{-3}{4(3x+5)\ln 10}$ (B) $\frac{1}{(3x+5)\ln 10}$ (C) $\frac{9}{4(3x+5)\ln 10}$ (D) $\frac{3}{4(3x+5)\ln 10}$

A ____ 10. If $y=\left(1+x^2\right)^x$, then $\frac{dy}{dx}=$

(A) $\left(1+x^2\right)^x\left[\frac{2x^2}{1+x^2}+\ln\left(1+x^2\right)\right]$ (B) $2x\left(1+x^2\right)^{x-1}$

(C) $\frac{2x^2}{1+x^2}+\ln\left(1+x^2\right)$ (D) $\left(1+x^2\right)^x\left[\frac{1}{1+x^2}+\ln\left(1+x^2\right)\right]$

Part II: Free Response—Show all work in a clear, concise, cogent, an complete manner

11. Let $f(x)=\begin{cases} 2e^{-x}-4, & x<0 \\ -2-2\sin x, & x\geq 0 \end{cases}$

(a) Show that f is differentiable at $x=0$.

Continuous

$\lim_{x\to 0^-} f(x) = 2e^{0}-4=-2$

$\lim_{x\to 0^+} f(x) = -2-2\sin 0=-2$

$f(0)=-2$

$\boxed{\text{Since } -2=-2=-2,}$ ✓1

$\boxed{f \text{ is continuous at } x=0}$ ✓2

$f'(x)=\begin{cases} -2e^{-x}, & x<0 \\ -2\cos x, & x>0 \end{cases}$ ✓3

$\lim_{x\to 0^-} f'(x) = -2e^{0}=-2$

$\lim_{x\to 0^+} f'(x) = -2\cos 0=-2$

$\boxed{\text{Since } -2=-2,}$ ✓4

$\boxed{f \text{ is differentiable @ } x=0.}$ ✓5

(b) Rolle's Theorem applies to $f(x)$ on the interval $\left[a,\frac{3\pi}{2}\right]$. If $a<0$, find the value of a.

$f(a)=f\left(\frac{3\pi}{2}\right)$

$2e^{-a}-4=-2-2\sin\frac{3\pi}{2}$

$2e^{-a}-4=-2-2(-1)$

$2e^{-a}-4=-2+2$

$\boxed{2e^{-a}-4=0}$ ✓6

$2e^{-a}=4$

$e^{-a}=2$

$\ln(e^{-a})=\ln(2)$

$-a=\ln 2$

$\boxed{a=-\ln 2 \text{ or } \ln\left(\tfrac{1}{2}\right)}$ ✓7

(c) Find the value(s) of c guaranteed by Rolle's Theorem on the interval $\left[a,\frac{3\pi}{2}\right]$. Justify.

$f'(x)=0$ for some $c\in\left(-\ln 2,\frac{3\pi}{2}\right)$ ✓8

So, either $-2e^{-x}=0$ on $(-\ln 2, 0)$ or $\boxed{-2\cos x=0}$ on $\left(0,\frac{3\pi}{2}\right)$

$e^{-x}=0$

$\ln e^{-x}=\ln 0$

$-x=\ln 0$

$x=-\ln 0$

$x=$ undefined

(no solution)

$\cos x=0$

$\boxed{x=\frac{\pi}{2}}$ ✓9