Name KEY Date Period

AB Calculus Practice Test: All Rules of Differentiation & Relative extrema

Part I: Multiple Choice

E

1. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2)=5$ and $f'(5)=-\frac{1}{2}$, then $g'(-2)=$

$g: (-2,5)$ $\quad$ $f: (5,-2)$ $\quad$ $g'(-2)=\frac{1}{f'(5)}=\frac{1}{-1/2}=\boxed{-2}$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

A

2. If f and g are twice differentiable and if $h(x)=f(g(x))$, then $h''(x)=$

(A) $f''(g(x))[g'(x)]^2+f'(g(x))g''(x)$

(B) $f''(g(x))g'(x)+f'(g(x))g''(x)$

(C) $f''(g(x))[g'(x)]^2$

(D) $f''(g(x))g''(x)$

(E) $f''(g(x))$

$h(x)=f(g(x))$

$h'(x)=f'(g(x))\cdot g'(x)$

$h''(x)=f''(g(x))\cdot g'(x)\cdot g'(x)+f'(g(x))\cdot g''(x)$

$h''(x)=f''(g(x))[g'(x)]^2+f'(g(x))g''(x)$

A

3. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

$\frac{d}{dx}[x^3+(3x)(y)+2y^3]=\frac{d}{dx}[17]$

$3x^2+3y+3x\frac{dy}{dx}+6y^2\frac{dy}{dx}=0$

$\frac{dy}{dx}(3x+6y^2)=-3x^2-3y$

$\frac{dy}{dx}=\frac{-\cancel{3}(x^2+y)}{\cancel{3}(x+2y^2)}$

E 4.

If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

$f'(x)=\cos\left(e^{-x}\right)\cdot\left(e^{-x}\right)\cdot(-1)$

$f'(x)=-e^{-x}\cos\left(e^{-x}\right)$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x})+e^{-x}$

(C) $\cos(e^{-x})-e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

C 5.

If $f(x)=(x-1)^{\frac{3}{2}}+\dfrac{e^{x-2}}{2}$, then $f'(2)=$

$f(x)=(x-1)^{3/2}+\frac{1}{2}e^{(x-2)}$

$f'(x)=\frac{3}{2}(x-1)^{1/2}\cdot(1)+\frac{1}{2}e^{(x-2)}\cdot(1)$

$f'(2)=\frac{3}{2}+\frac{1}{2}=\boxed{2}$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) $\dfrac{7}{2}$ (E) $\dfrac{3+e}{2}$

C 6.

$\dfrac{d}{dx}\left(xe^{\ln x^2}\right)=$ $\frac{d}{dx}\left(x\cdot x^2\right)=\frac{d}{dx}\left(x^3\right)=\boxed{3x^2}$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

A 7.

$\lim\limits_{h\to 0}\dfrac{\ln(e+h)-1}{h}$ is $\lim\limits_{h\to 0}\dfrac{\ln(e+h)-\ln e}{h}$, $f(x)=\ln x$, $f'(e)$

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

E 8.

If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

$f'(x)=\dfrac{(2x)(e^{2x})(2)-(e^{2x})(2)}{(2x)^2}$

$f'(x)=\dfrac{2e^{2x}[2x-1]}{4x^2}$ *factor out $2e^{2x}$

$f'(x)=\dfrac{e^{2x}(2x-1)}{2x^2}$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

D 9.

If $f(x)=\ln\left|x^2-1\right|$, then $f'(x)=$

$f'(x)=\dfrac{1}{(x^2-1)}(2x)$

$f'(x)=\dfrac{2x}{x^2-1}$

(A) $\left|\dfrac{2x}{x^2-1}\right|$

(B) $\dfrac{2x}{\left|x^2-1\right|}$

(C) $\dfrac{2|x|}{x^2-1}$

(D) $\dfrac{2x}{x^2-1}$

(E) $\dfrac{1}{x^2-1}$

B 10.

If $y=\arctan(e^{2x})$, then $\dfrac{dy}{dx}=$ $\dfrac{1}{1+(e^{2x})^2}\cdot e^{2x}\cdot 2=\dfrac{2e^{2x}}{1+e^{4x}}$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\dfrac{2e^{2x}}{1+e^{4x}}$ (C) $\dfrac{e^{2x}}{1+e^{4x}}$ (D) $\dfrac{1}{\sqrt{1-e^{4x}}}$ (E) $\dfrac{1}{1+e^{4x}}$

B 11. *Solve for f(x) 1st!!

$\ln(e^{f(x)}) = \ln(1+x^2)$

If $e^{f(x)} = 1+x^2$, then $f'(x) =$

$f(x) = \ln(1+x^2)$

$f'(x) = \left(\frac{1}{1+x^2}\right)(2x) = \frac{2x}{1+x^2}$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

A 12.

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

Since only x=1 is given, this a clue to solve for y 1st!

$e^{\ln(xy)} = e^{x}$, $xy = e^x$, $y = \frac{e^x}{x}$, $y' = \frac{(x)(e^x) - (e^x)(1)}{x^2}$, $y'(1) = \frac{e^1 - e^1}{1^2} = 0$

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

D 13.

If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

$f(x) = e^{(\tan x)^2}$

$f'(x) = e^{\tan^2 x} \cdot (2\tan x) \cdot \sec^2 x$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

B 14. $f(x) = 2x$, $f'(x) = 2$ *simplify early & often

If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

E 15.

If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$ $f(x) = e^{\ln(x^2)^3} = e^{\ln(x^6)} = x^6$, $f'(x) = 6x^5$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

C 16.

$\frac{d}{dx}\left(2^{x}\right)=$ $(2^x)(\ln 2)$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^{x})\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

E 17.

$\frac{d}{dx}\ln\left|\cos\left(\frac{\pi}{x}\right)\right|$ is $= \frac{1}{\cos\left(\frac{\pi}{x}\right)}\cdot\left(-\sin\frac{\pi}{x}\right)\cdot\left(-\frac{\pi}{x^2}\right) = \frac{\pi\sin\left(\frac{\pi}{x}\right)}{x^2\cos\left(\frac{\pi}{x}\right)} = \frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$

(A) $\frac{-\pi}{x^{2}\cos\left(\frac{\pi}{x}\right)}$ (B) $-\tan\left(\frac{\pi}{x}\right)$ (C) $\frac{1}{\cos\left(\frac{\pi}{x}\right)}$

(D) $\frac{\pi}{x}\tan\left(\frac{\pi}{x}\right)$ (E) $\frac{\pi}{x^{2}}\tan\left(\frac{\pi}{x}\right)$

B 18.

The slope of the line normal to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is

(A) -2

$y'=2\left(\frac{1}{\sec x}\right)(\sec x\tan x)$

$y'=2\tan x$

$y'\left(\frac{\pi}{4}\right)=2\tan\frac{\pi}{4}=2$

So normal slope is $\boxed{-\frac{1}{2}}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

A 19.

If $f(x)=e^{x}$, then $\ln\left(f'(2)\right)=$ $f'(x)=e^x$, $f'(2)=e^2$, $\ln(f'(2))=\ln e^2=\boxed{2}$

(A) 2 (B) 0 (C) $\frac{1}{e^{2}}$ (D) $2e$ (E) e^{2}

B 20.

If $f(x)=\ln\left(\sqrt{x}\right)$, then $f''(x)=$ $f(x)=\ln(x^{1/2})=\frac{1}{2}\ln x$, $f'(x)=\left(\frac{1}{2}\right)\left(\frac{1}{x}\right)=\frac{1}{2}x^{-1}$, $f''(x)=-\frac{1}{2}x^{-2}=\boxed{\frac{-1}{2x^2}}$

(A) $-\frac{2}{x^{2}}$ (B) $-\frac{1}{2x^{2}}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^{2}}$

E 21.

If $f(x)=\left(x^2+1\right)^x$, then $f'(x)=$

LOG DIFF

$\ln(f(x))=\ln(x^2+1)^x$

$\ln(y)=(x)\ln(x^2+1)$

$\frac{d}{dx}[\ln y]=\frac{d}{dx}[(x)(\ln(x^2+1))]$

$\left(\frac{1}{y}\right)\frac{dy}{dx}=(1)(\ln(x^2+1))+(x)\left(\frac{1}{x^2+1}\right)(2x)$

$\frac{dy}{dx}=\left[\ln(x^2+1)+\frac{2x^2}{x^2+1}\right](x^2+1)^x$

(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2-1\right)+\frac{2x^2}{x^2+1}$

(E) $\left(x^2-1\right)^x\left[\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}\right]$

A 22.

If $f(x)=\left(x^2+1\right)^{(2-3x)}$, then $f'(1)=$

$\boxed{\text{LOG DIFF}}$ $\ln(f(x))=(2-3x)\ln(x^2+1)$

$\frac{d}{dx}$: $\frac{1}{f(x)}\cdot f'(x)=(-3)(\ln(x^2+1))+(2-3x)\left(\frac{1}{x^2+1}\right)(2x)$

$f'(x)=\left(-3\ln(x^2+1)+\frac{2x(2-3x)}{x^2+1}\right)(x^2+1)^{(2-3x)}$

* $1=\ln e$

$f'(1)=\left(-3\ln 2+\frac{-2}{2}\right)(2^{-1})=\frac{1}{2}(-\ln 2^3-1)=-\frac{1}{2}(\ln 8+\ln e)=-\frac{1}{2}\ln(8e)$

(A) $-\frac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2}\ln(2)$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$

D 23.

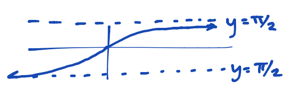
$\frac{d}{dx}(\arcsin(2x))=\frac{1}{\sqrt{1-(2x)^2}}(2)=\frac{2}{\sqrt{1-4x^2}}$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

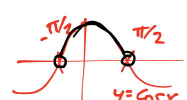
(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

B 24.

Let $f(x)=\cos(\arctan x)$. What is the range of f?



* Range of Arctangent is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and it becomes the domain for cos x.



Range of cos x for $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is $y\in(0,1]$

(A) $\left\{x \mid -\frac{\pi}{2}<x<\frac{\pi}{2}\right\}$ (B) $\{x \mid 0<x\le 1\}$ (C) $\{x \mid 0\le x\le 1\}$

(D) $\{x \mid -1<x<1\}$ (E) $\{x \mid -1\le x\le 1\}$

A

25.

If $y = e^{nx}$, then $\dfrac{d^n y}{dx^n} =$

$\frac{dy}{dx} = ne^{nx}$ *the n factor comes from the chain rule

$\frac{d^2y}{dx^2} = n^2e^{nx}$

$\frac{d^3y}{dx^3} = n^3e^{nx}$

$\vdots$

$\frac{d^ny}{dx^n} = n^n e^{nx}$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

A

26.

If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

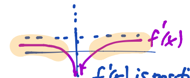
f increases when $f' > 0$

$(-\infty,-1)\cup(1,\infty)$ * certain schools of thought allow the endpts to be included.

$f'(x) = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} = 0$

$x = -1$ or $x = 1$



f'(x) is positive outside the x-intercepts.

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$

(D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

C

27.

If $y = x^{\ln x}$, then y' is

LOGDIFF: $\ln y = (\ln x)\ln x$

$\ln y = (\ln x)^2$

$\frac{d}{dx}[\ln y] = \frac{d}{dx}[(\ln x)^2]$

$\left(\frac{1}{y}\right)y' = 2(\ln x)^1\left(\frac{1}{x}\right)$

$y' = \left(\frac{2\ln x}{x}\right)x^{\ln x}$

(A) $\dfrac{x^{\ln x}\ln x}{x^2}$

(B) $x^{1/x}\ln x$

(C) $\dfrac{2x^{\ln x}\ln x}{x}$

(D) $\dfrac{x^{\ln x}\ln x}{x}$

(E) None of the above

C

28.

If $f(g(x)) = \ln(x^2+4)$, $f(x) = \ln(x^2)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\dfrac{1}{\sqrt{x^2+4}}$ (B) $\dfrac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

*composition $\ln\left(\left(\sqrt{x^2+4}\right)^2\right)$

$= \ln(x^2+4)$

D 29.

If $y=10^{(x^2-1)}$, then $\frac{dy}{dx}=$ $10^{(x^2-1)} \cdot \ln 10 \cdot (2x)$

(A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$

B 30.

An equation of the line tangent to $y=x^3+3x^2+2$ at its point of inflection is OMIT THIS ONE. THIS §5.4.

$y(-1)=-1+3+2=4$, $\boxed{pt\ (-1,4)}$

$y'(-1)=3-6=\boxed{-3=m}$

eq: $y=4-3(x+1)$

$y=4-3x-3$

$\boxed{y=-3x+1}$

$y'=3x^2+6x$

$y''=6x+6=0$

$\boxed{x=-1}$ y'' changes from pos to neg at $x=-1$ so $x=-1$ is an inflection value of $y=f(x)$.

(A) $y=-6x-6$ (B) $y=-3x+1$ (C) $y=2x+10$

(D) $y=3x-1$ (E) $y=4x+1$

D 31.

If f and g are twice differentiable functions such that $g(x)=e^{f(x)}$ and $g''(x)=h(x)e^{f(x)}$, then $h(x)=$

$g'(x)=e^{f(x)}\cdot f'(x)$

$g''(x)=(e^{f(x)}\cdot f'(x))(f'(x))+(e^{f(x)})(f''(x))=e^{f(x)}((f'(x))^2+f''(x))$

(A) $f'(x)+f''(x)$ (B) $f'(x)+(f''(x))^2$ (C) $(f'(x)+f''(x))^2$

(D) $(f'(x))^2+f''(x)$ (E) $2f'(x)+f''(x)$

E 32.

For $0<x<\frac{\pi}{2}$, if $y=(\sin x)^x$, then $\frac{dy}{dx}$ is

LOG DIFF $\ln y=(x)(\ln\sin x)$

$\frac{d}{dx}[\ln y]=\frac{d}{dx}[(x)(\ln\sin x)]$

$\left(\frac{1}{y}\right)(y')=(1)\ln\sin x+(x)\left(\frac{1}{\sin x}\right)(\cos x)$

$y'=(\ln\sin x+x\cot x)(\sin x)^x$

(A) $x\ln(\sin x)$ (B) $(\sin x)^x\cot x$ (C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x+\sin x)$ (E) $(\sin x)^x(x\cot x+\ln(\sin x))$

C 33.

$\frac{d}{dx}\left(x^{\ln x}\right)=$

LOG DIFF

$y=x^{\ln x}$

$\ln y=(\ln x)(\ln x)$

$\frac{d}{dx}[\ln y]=\frac{d}{dx}[(\ln x)^2]$

$\left(\frac{1}{y}\right)y'=2(\ln x)\left(\frac{1}{x}\right)$

$y'=\left(\frac{2\ln x}{x}\right)x^{\ln x}$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x-1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

BC Free Response:

1. 2001 AB4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

$$4 : \begin{cases} 1 : x = \pm\sqrt{2} \\ 1 : \text{analysis} \\ 2 : \text{conclusions} \\ \quad < -1 > \text{ not dealing with discontinuity at } 0 \end{cases}$$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

$$3 : \begin{cases} 1 : h''(x) \\ 1 : h''(x) > 0 \\ 1 : \text{answer} \end{cases}$$

(c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

1 : tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1 : answer with reason

2. 2008 AB6

Let f be the function given by $f(x)=\dfrac{\ln x}{x}$ for all $x>0$.

(a) Show that the derivative of f is given by $f'(x)=\dfrac{1-\ln x}{x^2}$.

(b) Write an equation for the line tangent to the graph of f at $x=e^2$.

(c) Find the x-coordinate of the point at which $f'(x)=0$.

(d) Find the x-coordinate of the point at which $f''(x)=0$.

(e) Find $\lim\limits_{x\to 0^+} f(x)$.

(a) $f(e^2)=\dfrac{\ln e^2}{e^2}=\dfrac{2}{e^2},\ f'(e^2)=\dfrac{1-\ln e^2}{\left(e^2\right)^2}=-\dfrac{1}{e^4}$ An equation for the tangent line is $y=\dfrac{2}{e^2}-\dfrac{1}{e^4}\left(x-e^2\right)$.	$2:\begin{cases}1: f(e^2) \text{ and } f'(e^2)\\ 1: \text{answer}\end{cases}$
(b) $f'(x)=0$ when $x=e$. The function f has a relative maximum at $x=e$ because $f'(x)$ changes from positive to negative at $x=e$.	$3:\begin{cases}1: x=e\\ 1: \text{relative maximum}\\ 1: \text{justification}\end{cases}$
(c) $f''(x)=\dfrac{-\frac{1}{x}x^2-(1-\ln x)2x}{x^4}=\dfrac{-3+2\ln x}{x^3}$ for all $x>0$ $f''(x)=0$ when $-3+2\ln x=0$ $x=e^{3/2}$ The graph of f has a point of inflection at $x=e^{3/2}$ because $f''(x)$ changes sign at $x=e^{3/2}$.	$3:\begin{cases}2: f''(x)\\ 1: \text{answer}\end{cases}$
(d) $\lim\limits_{x\to 0^+}\dfrac{\ln x}{x}=-\infty$ or Does Not Exist	1 : answer

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$ $\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$ $\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$	$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$
(b) $\left.\dfrac{dy}{dx}\right\rvert_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$ Tangent line: $y = 1 + \frac{1}{4}(x+2)$	$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$
(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$. On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$. Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.	$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$
(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$. The curve crosses the x-axis where $y = 0$. $(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$ No, the curve cannot have a horizontal tangent where it crosses the x-axis.	$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$