Name KEY ______________________ Date ______________________ Period ______________

Calculus Test: 4.1 to 5.1. No Calculator

Part I: Multiple Choice

A 1. If $f(x)=\ln\left|x+4+e^{-3x}\right|$, then $f'(0)=$ (A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) DNE

$\frac{1-3e^{-3x}}{x+4+e^{-3x}} \to \frac{-2}{5}$

B 2. Let f be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$? (A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

$f'=3x^2+1$ $f'(1)=4$

E 3. Find the global max and min of $f(x)=x^3-3x+1$ on the interval $[0,2]$.

(A) Global max at $x=0$; Global min at $x=1$

(B) Global max at $x=2$; Global min at $x=0$

(C) Global max at $x=2$, $x=-1$; Global min at $x=1$

(D) Global max at $x=1$; Global min at $x=1$

(E) Global max at $x=2$; Global min at $x=1$

$3x^2-3=0$ $x=-1,1$ $\boxed{x=1}$ $f(0)=1$ $f(1)=-1$ $f(2)=3$

B 4. The critical values of $f(x)=xe^{-x}$ are:

(A) $x=-1$ (B) $x=1$ (C) $x=1,\ x=-1$ (D) $x=0\ \ x=1$ (E) No critical values

$e^{-x}-xe^{-x}=e^{-x}(1-x)=0$

A 5. If $f(x)=x^2+2x$, then $\frac{d}{dx}\left[f(\ln x)\right]=$

$(\ln x)^2+2\ln x$

(A) $\frac{2\ln x+2}{x}$ (B) $2x\ln x+2$ (C) $2\ln x+2$ (D) $2\ln x+\frac{2}{x}$ (E) $\frac{2x+2}{x}$

$\frac{2\ln x}{x}+\frac{2}{x}$

A 6. The value of the derivative of $y=\frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x=0$ is

$\ln y=\frac{1}{3}\ln(x^2+8)-\frac{1}{4}\ln(2x+1)$

$y'=\left(\frac{1}{3}\left(\frac{2x}{x^2+8}\right)-\frac{1}{4}\left(\frac{2}{2x+1}\right)\right)y$

$\left(0-\frac{1}{2}\right)(2)$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

C 7. $\frac{d}{dx}\left[xe^{\ln x^2}\right]=$ (A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

A 8. $\lim\limits_{h\to 0}\frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$ (B) $f'(e)$, where $f(x)=\frac{\ln x}{x}$ (C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$ (E) $f'(0)$, where $f(x)=\ln x$

A 9. The slope of the line tangent to the graph of $\ln(xy)=x$ at the point where $x=1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

$xy=e^x$ $y=\frac{e^x}{x}$ $y'=\frac{xe^x-e^x}{x^2}$, $y'(1)=\frac{e-e}{1}=0$

E 10. If $f(x)=\left(x^2+1\right)^x$, then $f'(x)=$

(A) $x\left(x^2+1\right)^{x-1}$ (B) $2x^2\left(x^2+1\right)^{x-1}$ (C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}$ (E) $\left(x^2+1\right)^x\left[\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}\right]$

19 checks total

Part II: AB Free Response:

11. (1992 AB4/BC1) Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

1. A
2. B
3. E
4. B
5. A
6. A
7. C
8. A
9. A
10. E

$y + \cos y = x + 1, \quad 0 \le y \le 2\pi$

(a) $\frac{d}{dx}[y + \cos y] = \frac{d}{dx}[x+1]$

$\boxed{\frac{dy}{dx} - \sin y \frac{dy}{dx} = 1}$

$\frac{dy}{dx}(1 - \sin y) = 1$

$\boxed{\frac{dy}{dx} = \frac{1}{1-\sin y}}$

(b) Vert tangent when

$\frac{dy}{dx} = \frac{\neq 0}{0}$

$\boxed{1 - \sin y = 0}$

$\sin y = 1$

$\boxed{y = \frac{\pi}{2}}$

$\frac{\pi}{2} + \cos\frac{\pi}{2} = x + 1$

$\boxed{x = \frac{\pi}{2} - 1}$

$\boxed{9 \text{ checks}}$

(c) $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{1}{1-\sin y}\right]$

$\frac{d^2y}{dx^2} = \frac{-(-\cos y)\frac{dy}{dx}}{(1-\sin y)^2}$

$= \frac{\cos y}{(1-\sin y)^2}\left(\frac{1}{1-\sin y}\right)$

$= \boxed{\frac{\cos y}{(1-\sin y)^3}}$