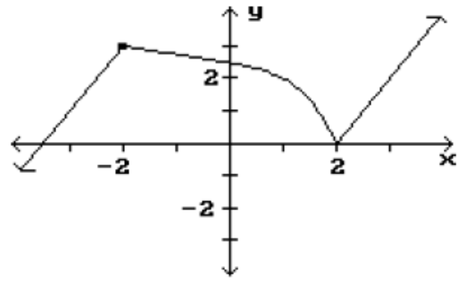
Name KEY Date $\boxed{\text{18 CHECKS TOTAL}}$ Period

.

Calculus Test: 4.1 to 5.1. No Calculator

Part I: Multiple Choice



A 1. Find the location and value of all relative extrema of the graph shown at right.

(A) Relative maximum of 3 at -2; Relative minimum of 0 at 2

(B) Relative maximum of 3 at -2. (C) Relative minimum of 0 at 2

(D) Relative maximum of -2 at 3; Relative minimum of 2 at 0. (E) None

B 2. The critical values of $f(x) = xe^{-x}$ are: $f' = e^{-x} - xe^{-x} = 0,\ e^{-x}(1-x) = 0$, $x = 1$

(A) $x = -1$ (B) $x = 1$ (C) $x = 1,\ x = -1$ (D) $x = 0\ \ x = 1$ (E) No critical values

E 3. Find the global max and min of $f(x) = x^3 - 3x + 1$ on the interval $[0,2]$. $f' = 3x^2 - 3 = 0,\ x = \pm 1$, only $x = 1 \in I$

(A) Global max at $x = 0$; Global min at $x = 1$ (B) Global max at $x = 2$; Global min at $x = 0$

(C) Global max at $x = 2$, $x = -1$; Global min at $x = 1$ (D) Global max at $x = 1$; Global min at $x = 1$

(E) Global max at $x = 2$; Global min at $x = 1$ $f(0) = 1,\ \underset{\text{MAX}}{f(2) = 3},\ \underset{\text{MIN}}{f(1) = -1}$

A 4. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0) =$ $f' = \dfrac{1 - 3e^{-3x}}{x + 4 + e^{-3x}},\ f'(0) = \dfrac{-2}{5}$

(A) $-\dfrac{2}{5}$ (B) $\dfrac{1}{5}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{2}{5}$ (E) nonexistent

A 5. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \dfrac{1}{4}$?

(A) 2 (B) $\dfrac{1}{2}$ (C) 0 (D) $-\dfrac{1}{2}$ (E) -2 $y' = \dfrac{4}{1 + 16x^2},\ y'\left(\tfrac{1}{4}\right) = \dfrac{4}{1+1} = 2$

B 6. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3,2)$?

(A) 0 (B) $\dfrac{4}{9}$ (C) $\dfrac{7}{9}$ (D) $\dfrac{6}{7}$ (E) $\dfrac{5}{3}$ $6y \cdot y' - 4x = -2y - 2xy'$, $12y' - 12 = -4 - 6y'$; $18y' = 8$, $y' = \dfrac{4}{9}$

B 7. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$? $g'(2) = \dfrac{1}{f'(1)},\ f' = 3x^2 + 1,\ f'(1) = 4$

(A) $\dfrac{1}{13}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{7}{4}$ (D) 4 (E) 13

A 8. If $f(x) = x^2 + 2x$, then $\dfrac{d}{dx}\left[f(\ln x)\right] =$ $f(\ln x) = (\ln x)^2 + 2(\ln x)$

$\dfrac{d}{dx}: \dfrac{2\ln x}{x} + \dfrac{2}{x} = \dfrac{2\ln x + 2}{x}$

(A) $\dfrac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \dfrac{2}{x}$ (E) $\dfrac{2x + 2}{x}$

E 9. If $y = x^2 \sin(2x)$, then $\dfrac{dy}{dx} =$ $y' = 2x\sin 2x + 2x^2\cos 2x$

$2x(\sin 2x + x\cos 2x)$

(A) $2x\cos(2x)$ (B) $4x\cos(2x)$ (C) $2x\left[\sin(2x) + \cos(2x)\right]$

(D) $2x\left[\sin(2x) - x\cos(2x)\right]$ (E) $2x\left[\sin(2x) + x\cos(2x)\right]$

Part II: AB Free Response:

10. (1992 AB4/BC1) Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\dfrac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

(a) $\frac{d}{dx}[y+\cos y] = \frac{d}{dx}[x+1]$

$\frac{dy}{dx} - \sin y \cdot \frac{dy}{dx} = 1$ ✓1 ✓2

$\frac{dy}{dx}(1-\sin y) = 1$

$\boxed{\frac{dy}{dx} = \frac{1}{1-\sin y}}$ ✓3

(b) Vertical tangent when

$\frac{dy}{dx} = \frac{\neq 0}{0}$ ✓4

$1-\sin y = 0$

$\sin y = 1$ ✓5

$\boxed{y = \frac{\pi}{2}}$

When $y = \frac{\pi}{2}$:

$\frac{\pi}{2} + \cos\frac{\pi}{2} = x+1$ ✓6

$x = \frac{\pi}{2} + 0 - 1$ ✓7

eq: $\boxed{x = \frac{\pi}{2} - 1 \text{ or } \frac{\pi-2}{2}}$

(c) $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[(1-\sin y)^{-1}\right]$ ✓8

$\frac{d^2y}{dx^2} = -(1-\sin y)^{-2}(-\cos y)\cdot\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = -(1-\sin y)^{-2}(-\cos y)\cdot\left(\frac{1}{1-\sin y}\right)$ ← ok

or $= \boxed{\frac{\cos y}{(1-\sin y)^3}}$ ✓9

or $\frac{d^2y}{dx^2} = \frac{(1-\sin y)(0) - (1)(-\cos y)\frac{dy}{dx}}{(1-\sin y)^2}$

$= \frac{\cos y\left(\frac{1}{1-\sin y}\right)}{(1-\sin y)^2}$

A
B
E
A
A
B
B
A
E