Name KEY Date___________ Favorite flavor of Strawberry______

AP Calculus TEST: 2.1 - 2.9 **NO CALCULATOR**

Part I: Mulitple Choice—Put the correct CAPITAL letter in the space to the left of each question.

D 1. What is the slope of the line tangent to the curve $y = \operatorname{arc\,cot}(5x)$ at the point at which $x = \frac{1}{10}$?

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) $-\frac{5}{4}$ (D) -4 (E) 4

$y' = \frac{-1}{1+25x^2}(5)$

$y' = \frac{-5}{1+\frac{25}{100}} = \frac{-5}{5/4} = -4$

B 2. If $f(x) = 3x^5 - 4x^4 + 7x^3 - e^x$, what is $\lim_{h \to 0} \frac{f^{(5)}(0+h) - f^{(5)}(0)}{h}$?

(A) 1 (B) -1 (C) 359 (D) 361 (E) 0

$f^{(6)} = 0 - e^x$

A 3. $\frac{d}{dx}\left[\sin^2\left(e^{3x}\right)\right] =$

(A) $6e^{3x}\sin\left(e^x\right)\cos\left(e^x\right)$ (B) $2e^{3x}\sin\left(e^x\right)\cos^2\left(e^x\right)$ (C) $6e^{3x}\sin\left(e^x\right)\cos^2\left(e^x\right)$

(D) $3e^{3x}\cos^2\left(e^x\right)$ (E) $2e^{3x}\cos^2\left(e^x\right)$

$2\sin\left(e^{3x}\right)\cdot\cos\left(e^{3x}\right)\cdot 3e^{3x}$

$6e^{3x}\sin\left(e^{3x}\right)\cos\left(e^{3x}\right)$

A 4. If $k(j(x)) = x = j(k(x))$, and if $k(-4) = 3$, $k(1) = -4$, $k'(-4) = -5$, $k'(1) = \frac{2}{5}$, find $j'(-4)$.

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) $-\frac{1}{5}$ (D) $\frac{1}{5}$ (E) $\frac{2}{5}$

$j(-4, 1)$

$k(1, -4)$

$= \frac{1}{k'(1)}$

$= \frac{5}{2}$

C 5. If $f(x) = \frac{3}{\ln 4} \cdot 2^x$, then $f'(4) =$

(A) 8 (B) 12 (C) 24 (D) $\frac{48}{\ln 4}$ (E) $\frac{48}{\ln 2}$

$f' = \frac{3}{\ln 4} \cdot 2^x \cdot \ln 2$

$f' = \frac{3\ln 2}{\ln 2^2} \cdot 2^x$

$f' = \frac{3\ln 2}{2\ln 2} \cdot 2^x$

$f' = \frac{3}{2} \cdot 2^x$

$f'(4) = \frac{3}{2}(2^4)$

$= \frac{3}{2}(16)$

$= 3 \cdot 8$

$= 24$

E 6. If $y = x^2 \sin(2x)$, then $\frac{dy}{dx} =$

(A) $2x\cos(2x)$ (B) $4x\cos(2x)$ (C) $2x[\sin(2x)+\cos(2x)]$

(D) $2x[\sin(2x)-x\cos(2x)]$ (E) $2x[\sin(2x)+x\cos(2x)]$

$y' = 2x\sin(2x) + x^2 \cdot \cos 2x \cdot 2$

$y' = 2x[\sin(2x) + x\cos(2x)]$

D 7. If $\lim\limits_{h\to 0} \frac{\arcsin(a+h)-\arcsin a}{h} = 2$ for some constant $a > 0$, then $a =$

$f'(a)$ //

for $f = \arcsin x$

$f' = \frac{1}{\sqrt{1-x^2}}$

$f'(a) = \frac{1}{\sqrt{1-a^2}}$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) 1

$\frac{1}{\sqrt{1-a^2}} = 2$

$\sqrt{1-a^2} = \frac{1}{2}$

$1 - a^2 = \frac{1}{4}$

$a^2 = \frac{3}{4}$ $a = \frac{\sqrt{3}}{2}$

B 8. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3,2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

y': $6yy' - 4x = -2y - 2xy'$

at $(3,2)$: $12y' - 12 = -4 - 6y'$

$18y' = 8$

$y' = \frac{4}{9}$

D 9. Let f be the function defined by $f(x) = 3x^5 + 2x^3 + 10x - 11$. Find $\left(f^{-1}\right)'(f(1))$.

(A) -31 (B) 31 (C) $-\frac{1}{31}$ (D) $\frac{1}{31}$ (E) $\frac{\ }{4}$

$f(1) = 3+2+10-11$

$f(1) = 4$

$f'(x) = 15x^4 + 6x^2 + 10$

$f'(1) = 31$

$(f^{-1})'(4) = \frac{1}{f'(1)} = \frac{1}{31}$

D 10. Find the x-values for which the graph of $2x^2 + 3xy + \frac{1}{2}y^2 = -5$ has vertical tangent lines.

I. $x = \sqrt{2}$

II. $x = -\sqrt{2}$

III. 0

y': $4x + 3y + 3xy' + y \cdot y' = 0$

$y' = \frac{-4x-3y}{3x+y}$

$y' = \text{DNE}$

$3x + y = 0$

$\boxed{y = -3x}$

So $2x^2 + 3x(-3x) + \frac{1}{2}(-3x)^2 = -5$

$2x^2 - 9x^2 + \frac{9}{2}x^2 = -5$

$-\frac{5}{2}x^2 = -5$

$x^2 = 2$

$x = \pm\sqrt{2}$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

Part II: Frei Response.

11. (AB 2000) #5

Consider the curve given by $6 + x^3 y = xy^2$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y^2 - 3x^2 y}{x^3 - 2xy}$

$\frac{d}{dx}$: $3x^2y + x^3\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx}$ ✓1

$\frac{dy}{dx}(x^3 - 2xy) = y^2 - 3x^2y$

$\frac{dy}{dx} = \frac{y^2 - 3x^2y}{x^3 - 2xy}$ ✓2

(b) Find every point(s) on the graph of the curve that has an x-coordinate of 1, then write an equation for the tangent line at every/each of these point(s).

when $x=1$: $6+y=y^2$ ✓3

$y^2-y-6=0$

$(y-3)(y+2)=0$

$y=3, y=-2$ ✓4

At $(1,3)$:

$\left.\frac{dy}{dx}\right|_{(1,3)}=0$

eq: $y=3+0(x-1)$ or $y=3$ ✓5

At $(1,-2)$

$\left.\frac{dy}{dx}\right|_{(1,-2)}=2$

eq: $y=-2+2(x-1)$ ✓6

(c) The graph of the curve has vertical tangent lines. Find the x-coordinate of each of these vertical tangent lines. Show the work that leads to your answer.

$\frac{dy}{dx}=\text{DNE}$

$2xy-x^3=0$ ✓7

$y=\frac{x^3}{2x}$

$y=\frac{1}{2}x^2$ (or $x=\pm\sqrt{2y}$)

when $y=\frac{1}{2}x^2$: $x\left(\frac{1}{2}x^2\right)^2-x^3\left(\frac{1}{2}x^2\right)=6$ ✓8 for substituting

$\frac{1}{4}x^5-\frac{1}{2}x^5=6$

$-\frac{1}{4}x^5=6$

$x^5=-24$

$x=\sqrt[5]{-24}$ ✓9