Name KEY Date 10/18/2017 Favorite Clown BOZO

AP Calculus TEST: 2.1-2.7, NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

E 1. If $f(x) = \dfrac{x^3 - c^3}{x^3 + c^3}$ where c is a constant, → so is c^3 then $f'(x) =$

(A) $\dfrac{-3c^3x^2}{(x^3+c^3)}$ (B) $\dfrac{-3c^3x^2}{(x^3+c^3)^2}$ (C) $\dfrac{3c^3x^2}{(x^3+c^3)^2}$ (D) $\dfrac{-6c^3x^2}{(x^3+c^3)^2}$ [✓ (E)] $\dfrac{6c^3x^2}{(x^3+c^3)^2}$

$$f'(x) = \frac{(x^3+c^3)(3x^2)-(x^3-c^3)(3x^2)}{(x^3+c^3)^2}$$

$$f'(x) = \frac{3x^2[x^3+c^3-x^3+c^3]}{(x^3+c^3)^2}$$

$$f'(x) = \frac{3x^2(2c^3)}{(x^3+c^3)^2} = \frac{6c^3x^2}{(x^3+c^3)^2}$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−2	1	13
2	−3	5	5	$-\frac{1}{2}$

The table above gives the values for differentiable functions $f(x)$ and $g(x)$ & their continuous derivatives at selected values. **Use the table to answer questions 2, 3, and 4.**

D 2. If $k(x) = f^2\left(\dfrac{x}{2}\right) =$, use the table to find $k'(2)$.

(A) −12 (B) −3 (C) 3 [✓ (D)] −6 (E) 6

$$k(x) = \left(f\left(\tfrac{1}{2}x\right)\right)^2$$

$$k'(x) = \cancel{2}\left(f\left(\tfrac{1}{2}x\right)\right)' \cdot f'\left(\tfrac{1}{2}x\right) \cdot \cancel{\tfrac{1}{2}}$$

$$\left\{ \begin{aligned} k'(2) &= f(1) \cdot f'(1) \\ &= (3)(-2) \\ &= -6 \end{aligned} \right.$$

E 3. If $J(x) = \sqrt{2f(x) + 3g(x)}$, find $J'(1)$.

(A) $\dfrac{315}{2}$ (B) $\sqrt{3}$ (C) 11 (D) $\dfrac{1}{2\sqrt{35}}$ [✓ (E)] $\dfrac{35}{6}$

$$J(x) = (2f(x)+3g(x))^{1/2}$$

$$J'(x) = \tfrac{1}{2}(2f(x)+3g(x))^{-1/2} \cdot (2f'(x)+3g'(x))$$

$$J'(1) = \tfrac{1}{2}(2f(1)+3g(1))^{-1/2} \cdot (2f'(1)+3g'(1))$$

$$\Bigg\} J'(1) = \frac{(2)(-2)+(3)(13)}{2\sqrt{2(3)+3(1)}} = \frac{-4+39}{2\sqrt{9}} = \frac{35}{6}$$

C 4. For $1 \le x \le 2$, which of the following must be true? $\boxed{\text{IVT QUESTION, D} \rightarrow \text{C}}$

I. $g(c) = \dfrac{7}{2}$ for some ✓ $c \in (1,2)$ $1 < 3.5 < 5$

II. $f(r) = 0$ for some ✓ $r \in (1,2)$ $-3 < 0 < 3$

III. $f(z) = 3.0001$ × for some $z \in (1,2)$

$g(1) = 1$, $g(2) = 5$

$f(1) = 3$, $f(2) = -3$

(takes on all y-values between endpt. y-values

(A) I only (B) II only [✓ (C)] I and II only (D) II and III only (E) I, II, and III

→ $3.001 \notin (-3, 3)$

C 5. $\lim\limits_{h\to 0}\dfrac{\dfrac{5}{\sqrt{(\cancel{7+h})-3}}-\dfrac{5}{\sqrt{7-3}}}{h}$ $= f'(7)$, for $f(x)=\frac{5}{\sqrt{x-3}}$

(A) $-\frac{5}{2}$ (B) $\frac{5}{2}$ [✓ (C) $-\frac{5}{16}$] (D) $\frac{5}{16}$ (E) DNE

$f=\frac{5}{\sqrt{x-3}}$

$f=5(x-3)^{-1/2}$

$f'=-\frac{5}{2}(x-3)^{-3/2}\cdot(1)$

$f'=-\frac{5}{2\sqrt{(x-3)^3}}$

$f'(7)=\frac{-5}{2(\sqrt{4})^3}$

$=\frac{-5}{16}$

A 6. If $f(x)=\cos x$ and $\frac{2}{y}=f(x)$, find $\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{6}}$

[✓ (A) $\frac{4}{3}$] (B) $4\sqrt{3}$ (C) $-4\sqrt{3}$ (D) $\frac{8}{\sqrt{3}}$ (E) $\frac{-8}{\sqrt{3}}$

So, $\cos x=\frac{2}{y}$

$y=\frac{2}{\cos x}$

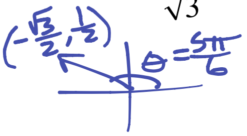
$y=2\sec x$

$\frac{dy}{dx}=2\sec x\cdot\tan x$

$\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{6}}=2\sec\frac{5\pi}{6}\tan\frac{5\pi}{6}$

$=2\left(-\frac{2}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)$

$=\frac{4}{3}$



A 7. If $f(1)=\frac{\pi}{4}$ and $f'(1)=3$, find the equation of the tangent line to $h(x)=\cot(f(x))$ at $x=1$.

[✓ (A) $6x+y=7$] (B) $6x-y=7$ (C) $6x-y=-7$ (D) $-6x+y=2$ (E) $6x-y=7$

pt: $(1,1)$

$m=-6$

eq: $y=1-6(x-1)$

$y=1-6x+6$

$6x+y=7$

$h(1)=\cot(f(1))$

$=\cot\frac{\pi}{4}$

$=1$

$h'(x)=-\csc^2(f(x))\cdot f'(x)$

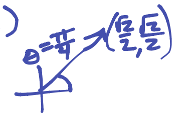
$h'(1)=-\left[\csc(f(1))\right]^2\cdot f'(1)$

$=-\left[\csc\frac{\pi}{4}\right]^2\cdot 3$

$=-3\left(\frac{2}{\sqrt{2}}\right)^2$

$=-3\left(\frac{4}{2}\right)$

$=-6$



C 8. $\frac{d^{41}}{dx^{41}}\left[\cos 2x+\underbrace{22x^{30}-42x^{20}+100x^{10}-99}_{\text{all goes to zero at 31st derivative}}\right]=$

(A) $2^{41}\sin 2x$ 10 (B) $2^{41}\cos 2x$ [✓ (C) $-2^{41}\sin 2x$] (D) $-2^{41}\cos 2x$ (E) $-82\sin 2x$

$4\overline{)41}$
$\underline{40}$
R1

So $\frac{d^{41}}{dx^{41}}=\frac{d}{dx}$ (1st deriv)

$y=\cos 2x$

$\frac{dy}{dx}=-2^1\sin 2x$, so, $\frac{d^{41}}{dx^{41}}[\cos 2x]=-2^{41}\cdot\sin 2x$

A 9. If $x^2+xy+y^3=0$, then in terms of x and y, $\frac{dy}{dx}=$

[✓ (A) $-\frac{2x+y}{x+3y^2}$] (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

$\frac{d}{dx}\left[x^2+xy+y^3\right]=\frac{d}{dx}[0]$

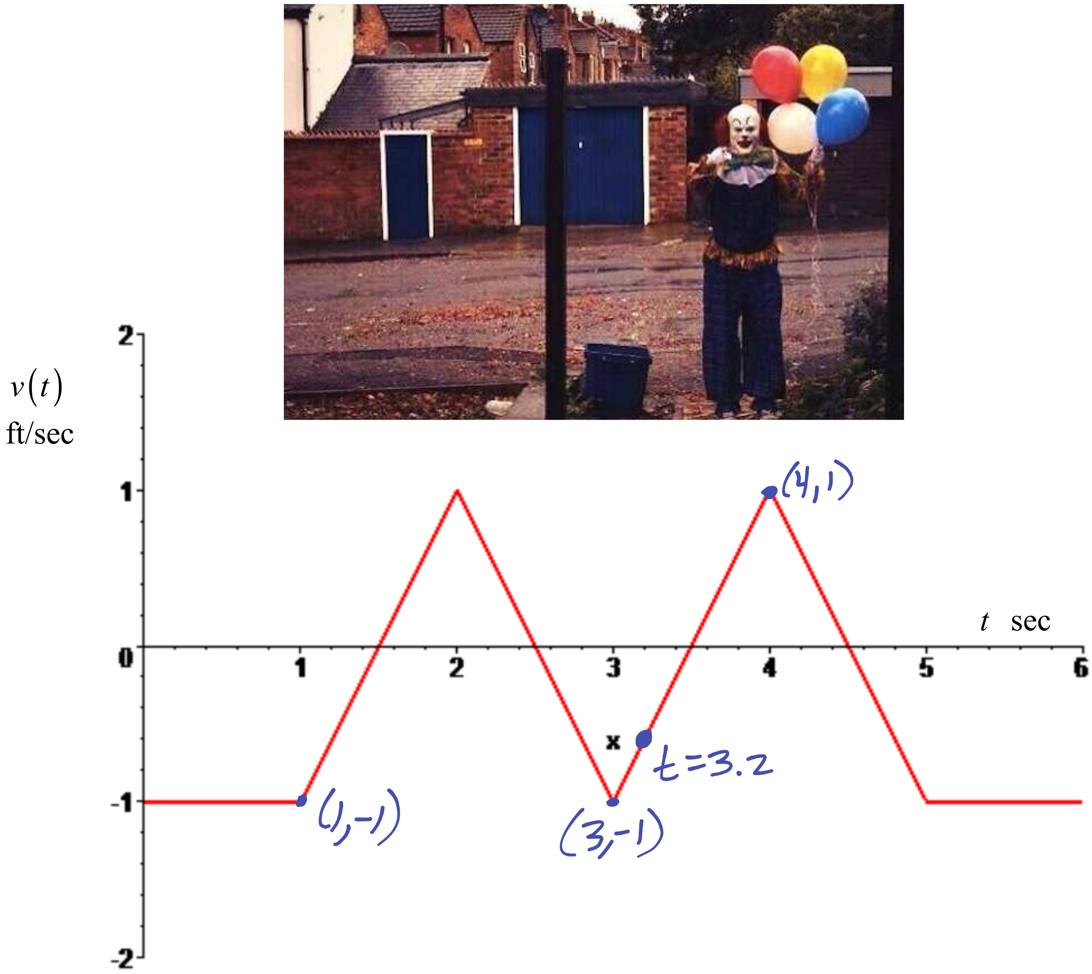
$2x+(1)(y)+(x)\left(\frac{dy}{dx}\right)+3y^2\left(\frac{dy}{dx}\right)=0$

$\frac{dy}{dx}\left(x+3y^2\right)=-2x-y$

$\frac{dy}{dx}=-\frac{2x+y}{x+3y^2}$

Part II: Free Response—Show all set ups, use correct notation, indicate your methods, and answer in complete math/English sentences (with units) when appropriate.

10. A creepy clown is walking along a sidewalk. His velocity, in ft/sec, is given as a function of time, in seconds, by the graph below for $0 \le t \le 6$.



(a) At $t = 3.2$ seconds, what is the clown's acceleration? Show the work that leads to your answer and answer with correct units. Write a sentence, with units, describing what your answer means in terms of the clown's velocity.

$v'(3.2) = a(3.2) = \frac{1-(-1)}{4-3} = \frac{2}{1} = 2 \text{ ft/sec}^2$ √1

At t=3.2 seconds, the clown's velocity is INCREASING by 2 ft/sec per second. √2

(b) What is the clown's average acceleration for $1 \le t \le 4$ seconds. Show the work that leads to your answer. Use proper units.

$$\text{Avg accel} = \frac{V(4)-V(1)}{4-1}$$ ✓3 Difference Quotient

$$= \frac{1-(-1)}{4-1}$$ or

$$= \frac{2}{3}\ \text{ft/sec}^2$$ ✓4

(c) At $t = 3.2$ seconds, is the speed of the clown increasing or decreasing? Justify.

Justification

① $V(3.2) < 0$

Slopes of graph of $v(t) > 0$ at $t = 3.2$ seconds.

✓5

OR ② $V(3.2) < 0$ and increasing.

or ③ The graph of $v(t)$ is approaching the t-axis at $t = 3.2$ seconds

So, speed is decreasing. ✓6

(d) On the interval $0 < t < 6$ seconds. How many times does the clown change direction. Explain how you know this.

✓4 4 times, because the graph of $v(t)$ changes from pos to neg twice and neg to pos twice for $0 < t < 6$ seconds.

✓8

units on (a) & (b) ✓9