Name_____ Date_____ Favorite Cold Yam Temperature_____
AP Calculus TEST: 2.1-2.7, NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question. SHOW ALL WORK/INDICATE ALL METHODS on a SEPARATE SHEET OF PAPER. Number your scratch work.

_____1. $\lim_{h\to 0}\dfrac{\dfrac{8}{\sqrt[3]{(6+h)+2}}-\dfrac{8}{\sqrt[3]{6+2}}}{h}=$

(A) $-\dfrac{1}{6}$ (B) $\dfrac{1}{6}$ (C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{2}$ (E) DNE

_____ 2. If $f(x)=\dfrac{x^2+c^2}{x^2-c^2}$ where c is a constant, then $f'(x)=$

(A) $\dfrac{-4c^2x}{\left(x^2-c^2\right)^2}$ (B) $\dfrac{-2c^2x}{\left(x^2-c^2\right)^2}$ (C) $\dfrac{4c^2x}{\left(x^2-c^2\right)}$ (D) $\dfrac{2c^2x}{\left(x^2-c^2\right)^2}$ (E) 1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−2	$\frac{5}{2}$	13
2	−3	5	5	$-\frac{1}{2}$

The table above gives the values for differentiable functions $f(x)$ and $g(x)$ & their continuous derivatives at selected values. **Use the table to answer questions 3, 4, and 5.**

_____ 3. If $h(x)=\left[f(2x)\right]^3$, use the table to find $h'(1)$.

(A) 54 (B) −90 (C) 135 (D) 270 (E) $\dfrac{10}{3}$

_____ 4. If $K(x)=\sqrt[3]{f(x)+2g(x)}$, find $K'(1)$.

(A) −2 (B) 2 (C) $-\dfrac{3}{2}$ (D) $\dfrac{3}{2}$ (E) 0

_____ 5. For $1\le x\le 2$, which of the following must be true?

I. $g(c)=\dfrac{7}{2}$ for some $c\in(1,2)$

II. $g'(r)=0$ for some $r\in(1,2)$

III. $f(a)=g(a)$ for some $a\in(1,2)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 6. If $f(x)=\cot x$ and $\frac{3}{y}=f(x)$, find $\left.\frac{dy}{dx}\right|_{x=\frac{11\pi}{6}}$

(A) 0 (B) 4 (C) 12 (D) $3\sqrt{3}$ (E) $-\sqrt{3}$

_____ 7. Let $f(x)=2\sin x\cos x$ for $0\le x<\pi$. Find all the values for which $f'(x)=1$.

I. $x=\frac{\pi}{6}$

II. $x=\frac{5\pi}{6}$

III. $x=\frac{\pi}{3}$

(A) I only (B) II only (C) I, II only (D) I, III only (E) II, and III only

_____ 8. If $h(x)=x^2g'(x)$, where $g(x)=\frac{x+\sec x}{x}$, then what is the slope of the secant line on the graph of $h(x)$ for $x\in[\pi,2\pi]$?

(A) -2 (B) 2 (C) $-\frac{2}{\pi}$ (D) $\frac{2}{\pi}$ (E) 0

_____ 9. If $\frac{d}{dx}\left[\left(\frac{x+3}{2x-1}\right)^4\right]=$

(A) $-28\frac{(x+3)^3}{(2x-1)^3}$ (B) $-28\frac{(x+3)^3}{(2x-1)^5}$ (C) $28\frac{(x+3)^3}{(2x-1)^5}$ (D) $-20\frac{(x+3)^3}{(2x-1)^3}$ (E) $20\frac{(x+3)^3}{(2x-1)^5}$

_____ 10. If $f(1)=\frac{\pi}{4}$ and $f'(1)=3$, find the equation of the tangent line to $h(x)=\cot(f(x))$ at $x=1$.

(A) $6x-y=7$ (B) $6x+y=7$ (C) $6x-y=-7$ (D) $-6x+y=2$ (E) $6x-y=7$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-3	2	-1	$\frac{1}{2}$
2	1	-2	3	$\frac{1}{4}$
3	5	$4a$	0	$\frac{1}{6}$

_____ 11. The table above gives the values for differentiable functions $f(x)$ and $g(x)$ & their derivatives at selected values. Find the value of a (if it exists) so that the tangent lines to $f(g(x))$ and $g(f(x))$ are perpendicular at $x=2$.

(A) $-\frac{1}{2}$ (B) 1 (C) 2 (D) -5 (E) DNE

_____ 12. $\frac{d^{50}}{dx^{50}}[\sin 3x] =$

(A) $3^{50}\sin 3x$ (B) $3^{50}\cos 3x$ (C) $-3^{50}\sin 3x$ (D) $-3^{50}\cos 3x$ (E) $150\cos 3x$

_____ 13. Find the slope of the tangent line to $x^2y^2 = (x+2)^2(40-y^2)$ at $(-3,2)$.

(A) $-\frac{6}{5}$ (B) $\frac{3}{5}$ (C) $\frac{9}{10}$ (D) $-\frac{3}{10}$ (E) 4

_____ 14. If $\cos(xy) = \frac{\sqrt{3}}{2}$, find $\frac{d^2y}{dx^2}$.

(A) $\frac{1}{\sin(xy)}$ (B) $-\frac{1}{\sin(xy)}$ (C) 0 (D) $\frac{2y}{x^2}$ (E) $-\frac{2y}{x^2}$

_____ 15. When the gates opened at the Unicorn football game, everyone rushed in. The rate R at which rabid fans entered Unicorn stadium at different times is given in the table below. Based on the data below, which of the following statements can be quantitatively substantiated?

time (sec)	0	25	50	75	100	180
R (people/sec)	70	120	220	340	300	160

I. At $t=2$ seconds, the instantaneous rate of change of R is approximately 2 people/sec^2

II. At $t=24$ seconds, the instantaneous rate of change of R is approximately 8 people/sec^2

III. At $t=75$ seconds, more people are entering Unicorn stadium that at any other time during the first 1.5 minutes.

(A) I only (B) II only (C) I and III only (D) I and II only (E) I, II, and III

Part II: Free Response—Show all set ups, use correct notation, indicate your methods, and answer in complete math/English sentences (with units) when appropriate.

16. An elephant moves along the *x*-axis so that at any time $t \in [0, 2\pi]$ seconds, its position, in feet, is given by $x(t) = 2t\sin t + 2\cos t + t^2$.

(a) Determine if the speed of the elephant is increasing or decreasing at $t = \frac{\pi}{6}$ seconds. Justify your answer.

(b) For what value(s) of $t \in (0, 2\pi)$ is the elephant at rest? Show the work that leads to your answer.

(c) On the interval $0 \le t \le 2\pi$ seconds, how far does the elephant travel? Justify your answer.