Name KEY ____________________ Date__________ Favorite Cold Yam Temperature_______

AP Calculus TEST: 4.1-4.7, NO CALCULATOR

Part Eins: Vielen choices—Put the correct CAPITAL letter in the space to the left of each question.

$\cot 2x \to -\csc^2(2x)\cdot 2 = -2(\csc(2x))^2$ $\quad$ @ $\frac{5\pi}{6}$ $-2(\csc(\frac{5\pi}{3}))^2 = -2(-\frac{2}{\sqrt{3}})^2 = -2(\frac{4}{3}) = -8/3$

__A__ 1. $\lim_{h\to 0} \dfrac{\cot 2\left(\frac{5\pi}{6}+h\right)-\cot\frac{5\pi}{3}}{h} =$

(A) $-\frac{8}{3}$ (B) $\frac{8}{3}$ (C) $-\frac{4}{3}$ (D) $\frac{4}{3}$ (E) $-\frac{3}{2}$

__C__ 2. If $y=\dfrac{x-1}{x+1}$, then $\dfrac{dy}{dx}=$ $\quad$ $y'=\dfrac{(x+1)(1)-(x-1)(1)}{(x+1)^2} = \dfrac{x+1-x+1}{} = \dfrac{2}{(x+1)^2}$

(A) $\frac{2x}{(x+1)^2}$ (B) $\frac{2}{x+1}$ (C) $\frac{2}{(x+1)^2}$ (D) $-\frac{2x}{(x+1)^2}$ (E) $\frac{2x}{x+1}$

__E__ 3. If f is differentiable at $x=0$, and $g(x)=[f(x)]^2$, $f(0)=f'(0)=-1$, then $g'(0)=$

(A) -2 (B) -1 (C) 1 (D) 4 (E) 2

$g'(x)=2(f(x))'\cdot f'(x)$
$g'(0)=2f(0)\cdot f'(0)=2(-1)(-1)$

__B__ 4. Suppose $x^2-xy+y^2=3$, find $\dfrac{dy}{dx}$ at the point (a,b).

(A) $\frac{a-2b}{2a-b}$ (B) $\frac{b-2a}{2b-a}$ (C) $\frac{a-2b}{2a+b}$ (D) $\frac{b-2a}{2b+a}$ (E) $\frac{b+2a}{2b+a}$

$2x-y-xy'+2yy'=0$
$2a-b-ay'+2by'=0$
$y'(-a+2b)=b-2a$
$y'=\frac{-2a+b}{-a+2b}=\frac{b-2a}{2b-a}=\frac{2a-b}{a-2b}$

__C__ 5. If $\sin y=\cos x$, find $\dfrac{dy}{dx}$ at the point $\left(\frac{\pi}{2},\pi\right)$

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{2}$ (E) none of these

$\cos y\cdot y'=-\sin x$, $\cos\pi\cdot y'=-\sin\frac{\pi}{2}$
$-y'=-1$, $\boxed{y'=1}$

__C__ 6. An equation of the line tangent to the graph of $y=x^2(2x+1)^4$ at $x=-1$. $\quad y(-1)=1 \quad (-1,1)$

(A) $y=-6x-5$ (B) $y=-6x+2$ (C) $y=-10x-9$ (D) $y=-10x+11$ (E) $y=6x+7$

$2x(2x+1)^4+x^2(4)(2x+1)^3(2)$
@ $x=-1$: $-2(1)+4(-1)(2)=-2-8=\boxed{-10}$

$y=1-10(x+1)$
$y-1=-10x-10$
$y=-10x-9$

__C__ 7. $\dfrac{d}{dx}[\csc x-\cos x]=$

(A) $\csc x\cot x-\sin x$ (B) $-\csc^2 x-\sin x$ (C) $\sin x-\csc x\cot x$ (D) $-\csc^2 x+\sin x$ (E) 5

$-\csc x\cot x+\sin x$

__D__ 8. If $y=-\dfrac{1}{\sqrt{x^2+1}}$, then $\dfrac{dy}{dx}=$

$y=-(x^2+1)^{-1/2}$, $y'=\frac{1}{2}(x^2+1)^{-3/2}(2x)$
$y'=\frac{x}{\sqrt{(x^2+1)^3}}$

(A) $\frac{x}{\sqrt{x^2+1}}$ (B) $-\frac{x}{\sqrt{x^2+1}}$ (C) $-\frac{x}{\sqrt{(x^2+1)^3}}$ (D) $\frac{x}{\sqrt{(x^2+1)^3}}$ (E) $\frac{x}{x^2+1}$

Part Los Dos: Frei Response.

9. An elephant moves along a vertical line and has a position equation $y(t)=\left(3t \quad 1\right)\left(t \quad 2\right)$ with $y(t)$ measured in furlongs (about 210 meters) and t measured in heleks (about 3.3 seconds) and $t \geq 0$. Answer the following. Be sure to include units in your final answer(s), lest you lose valuable points and class rank slots.

$y(t)= 3t^2-7t+2$

(a) What is the initial position of the elephant?
(b) When is the first time the elephant is at the zero position?
(c) What is the elephant's displacement on the interval from $t=0$ to $t=1$ heleks? Explain what that number means in terms of the elephant's starting position.
(d) What is the elephant's average velocity on the interval from $t=0$ to $t=1$ heleks?
(e) What is the elephant's velocity at $t=1$ heleks? Write a sentence explaining the meaning of your answer in terms of the elephant's position.
(f) What is the elephant's acceleration at $t=1$ heleks?
(g) At what time (in heleks) does the elephant change directions? Justify.
(h) At $t=1$ heleks, is the speed of the elephant increasing or decreasing? Justify.

(a) $y(0)=2$ furlongs

(b) $(3t-1)(t-2)=0$

$t=\frac{1}{3}, t=2$

$t=\frac{1}{3}$ heleks

(c) $y(1)-y(0)$

$=(2)(-1)-2$

$=-4$ furlongs

*On the interval from $t=0$ to $t=1$ heleks, the elephant ended up 4 furlongs Below where it started.

(d) Avg $=\dfrac{y(1)-y(0)}{1-0}$

$\dfrac{-4}{1}=-4$ furlongs/helek

(e) $y'(t)=v(t)=6t-7$

$v(1)=-1$ furlong/helek

At $t=1$ helek, the elephant is moving Down at 1 furlong/helek

(f) $y''(t)=v'(t)=a(t)=6$

$a(1)=6$ furlongs/helek^2

(g) When $y'(t)=v(t)$ changes signs

$v(t)=6t-7=0$, $t=\frac{7}{6}$ heleks

Since $v(t)$ changes from negative to positive @ $t=7/6$ helek.

(h) $a(1)=6>0$ So at $t=1$ heleks,
$v(1)=-1<0$ Speed is Decreasing