Name KEY Date Thur, 10-10-2018 Favorite Bear Kodiak Grizzly Pooh Bear

BC Calculus TEST: 2.1 - 2.6, NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

A 1. If $f(x) = \sec x$, find $\lim_{h \to 0} \dfrac{f'(\pi + h) - f'(\pi)}{h}$. $= f''(\pi)$

(A) -1 (B) 0 (C) 1 (D) 2 (E) -2

$f'(x) = \sec x \cdot \tan x$

$f''(x) = (\sec x \tan x) \cdot \tan x + \sec x \cdot (\sec^2 x)$

$f''(x) = \sec x \cdot \tan^2 x + \sec^3 x$

$f''(\pi) = \sec\pi \cdot (\tan\pi)^2 + (\sec\pi)^3$

$= (-1) \cdot (0^2) + (-1)^3$

$= 0 - 1 = -1$

B 2. If $f(x) = \sqrt[3]{x} \cot x$, what is $f'(x)$?

(A) $\dfrac{\cot x + 3x\csc^2 x}{3\sqrt[3]{x^2}}$ (B) $\dfrac{\cot x - 3x\csc^2 x}{3\sqrt[3]{x^2}}$ (C) $\dfrac{3x\csc^2 x - \cot x}{3\sqrt[3]{x^2}}$

(D) $\dfrac{-\csc x \cot x}{3\sqrt[3]{x^2}}$ (E) $\dfrac{\csc x \cot x}{3\sqrt[3]{x^2}}$

product rule

$f(x) = (x^{1/3})(\cot x)$

$f'(x) = \left(\frac{1}{3}x^{-2/3}\right)(\cot x) + (x^{1/3})(-\csc^2 x)$

$f'(x) = \frac{1}{3}x^{-2/3}(\cot x - 3x\csc^2 x)$

$f'(x) = \dfrac{\cot x - 3x\csc^2 x}{3\sqrt[3]{x^2}}$

D 3. If $f(x) = \begin{cases} 2ax^2 + x + 2, & x < -1 \\ bx + 3, & x \ge -1 \end{cases}$, what is the value of b that makes $f(x)$ differentiable at $x = -1$?

(A) -1 (B) 1 (C) -3 (D) 3 (E) $-\frac{1}{2}$

continuity

$\lim_{x \to -1^-} f(x) = 2a - 1 + 2$

$= 2a + 1$

$f(-1) = -b + 3$

$\lim_{x \to -1^-} f(x) = -b + 3$

So, $2a + 1 = -b + 3$

$b = 2 - 2a$

$f'(x) = \begin{cases} 4ax + 1, & x < -1 \\ b, & x > -1 \end{cases}$

slopes

$\lim_{x \to -1^-} f'(x) = -4a + 1$

$\lim_{x \to -1^+} f'(x) = b$

So, $b = -4a + 1$

So, $2 - 2a = -4a + 1$

$2a = -1$

$\boxed{a = -\frac{1}{2}}$

and $b = 2 - 2(-\frac{1}{2})$ plug back in

$b = 2 + 1$

$\boxed{b = 3}$

A 4. A particle (named Happy Joseph) moves along the x-axis such that at any time $\boxed{t \ge 0}$, its position function is given by $x(t) = t^3 - 9t^2 + 15t + 2$. On what intervals of t is Happy Joseph moving right?

(A) $[0,1) \cup (5,\infty)$ (B) $(1,5)$ (C) $(5,\infty)$ (D) $[0,1)$ (E) $(1,\infty)$

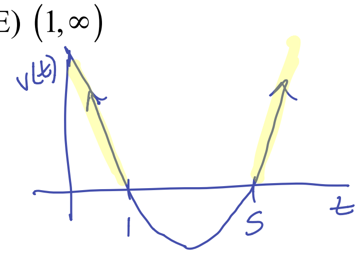
We want to know when $x'(t) = v(t) > 0$

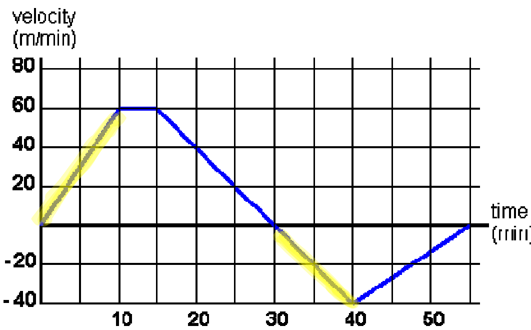
$x'(t) = v(t) = 3t^2 - 18t + 15 > 0$

$3(t^2 - 6t + 5) > 0$

$3(t - 5)(t - 1) > 0$

So, $t \in [0,1) \cup (5,\infty)$





E 5. The graph of the velocity of an indecisive tortoise (in meters per minute) is given above as it moves along a horizontal line. It's time is measured in minutes. On what intervals of t is the speed of the tortoise increasing?

- Speed increases when $v(t)$ and $v'(t)=a(t)$ are same sign.
- this is when the y-values & slopes of $v(t)$ are both positive or both negative.
- This is also when the graph of $v(t)$ moves AWAY from the t-axis (x-axis).

this is on $t \in (0,10)$ min and $t \in (30,40)$ min

√I. $(0,10)$ minutes

II. $(40,50)$ minutes speed dec

√III. $(30,40)$ minutes

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

E 6. If $f(x)=\tan^4(3x)$, that is $f'\left(\frac{5\pi}{12}\right)$?

(A) -6 (B) -12 (C) -24 (D) 12 (E) 24

Chain Rule

$f(x)=(\tan(3x))^4$ (3 layers means 3 factors)

$f'(x)=4(\tan(3x))^3 \cdot \sec^2(3x)\cdot 3$

$f'(x)=12\tan^3(3x)\cdot\sec^2(3x)$

$f'(\frac{5\pi}{12})=12(\tan(3\cdot\frac{5\pi}{12}))^3(\sec(3\cdot\frac{5\pi}{12}))^2$

$=12(\tan\frac{5\pi}{4})^3(\sec\frac{5\pi}{4})^2$

$\theta=\frac{5\pi}{4}$ $(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$

$f'(\frac{5\pi}{12})=12(1)^3(-\sqrt{2})^2$

$=12(2)$

$=24$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	-4	8
2	-1	-3	5	-5
3	3	-3	6	5
4	-2	4	7	2
5	-4	-8	3	-1

The table above gives selected values of differentiable functions $f(x)$ and $g(x)$ and their derivatives. Use this table to answer questions 7, 8, and 9.

C 7. If $h(x)=f(2x)\cdot g(x)$, what is $h'(2)$?

(A) 30 (B) 40 (C) 50 (D) -20 (E) -40

$h'(x)=2f'(2x)\cdot g(x)+f(2x)\cdot g'(x)$

$h'(2)=2f'(4)g(2)+f(4)\cdot g'(2)$

$h'(2)=(2)(4)(5)+(-2)(-5)$

$h'(2)=40+10=50$

A 8. If $K(x)=[f(g(x))]^3$, what is $K'(5)$?

(A) 81 (B) -81 (C) 27 (D) -27 (E) -3

$K'(x)=3[f(g(x))]^2\cdot f'(g(x))\cdot g'(x)$

$K'(5)=3[f(g(5))]^2\cdot f'(g(5))\cdot g'(5)$

$=3[f(3)]^2\cdot f'(3)\cdot(-1)$

$K'(5)=3[3]^2\cdot(-3)(-1)$

$K'(5)=27(3)$

$K'(5)=81$

A 9. If $J(x)=\dfrac{g(x)}{f(x)}$, what is $J'(1)$?

(A) 5 (B) 3 (C) 10 (D) 6 (E) 7

$J'(x)=\dfrac{f(x)g'(x)-g(x)f'(x)}{[f(x)]^2}$

$J'(1)=\dfrac{f(1)g'(1)-g(1)f'(1)}{[f(1)]^2}$

$J'(1)=\dfrac{(2)(8)-(-4)(1)}{2^2}$

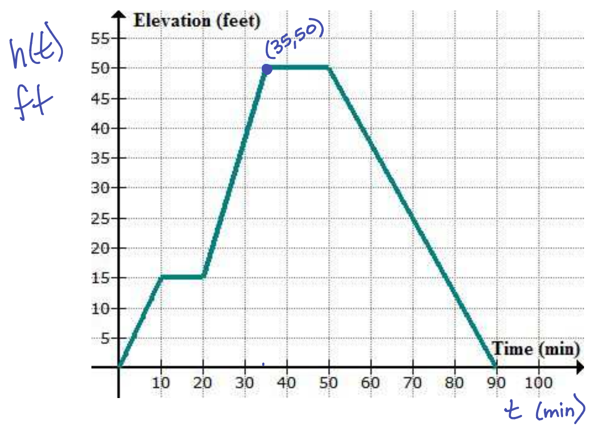
$J'(1)=\dfrac{16+4}{4}$

$J'(1)=\dfrac{20}{4}$

$J'(1)=5$

Part II: Free Response—show all work in a detailed, logical manner in the space provided. Include units on all final numeric answers.

10. Winnie-the-Pooh is climbing up and down on a tree. His elevation $h(t)$ above ground (in feet) at time $t \geq 0$ minutes is given in the graph below.



(a) On what open intervals is Pooh climbing up? Include units.

$t \in (0,10) \cup (20,35)$ minutes

✓1

(b) On what open intervals is Pooh at rest? Include units.

$t \in (10,20)$ minutes

✓2

(c) At what time does Pooh begin to descend? Include units.

@ $t = 50$ min

✓3

(d) What is Pooh's average velocity during the first 50 minutes. Show the work that leads to your answer. Include units.

$$\text{Avg velocity} = \frac{h(50)-h(0)}{50-0}$$

$$= \frac{50-0}{50-0}$$

$$= 1 \text{ ft/min}$$

✓4 in the prensence of the difference quotient

(e) What is Pooh's velocity at $t = 8$ minutes. Include units. Explain your answer in a complete sentence, with units, in terms of Pooh's elevation.

$h'(8) = v(8) = \frac{3}{2}$ or 1.5 ft/min ✓5

using pts $(0,0)$ & $(10,15)$ to find the slope of the line segment for $t \in (0,10)$

$m = \frac{15-0}{10-0}$

$m = \frac{15}{10} = \frac{3}{2} = 1.5$

At $t = 8$ minutes, Pooh's elevation was increasing by 1.5 feet each minute. ✓6

(f) On what open interval is Pooh's speed the slowest? Include units.

when he is not moving.

$t \in (10, 20)$ min ✓7

(g) What is the total distance Pooh travels vertically on $0 \le t \le 90$ minutes? Include units.

Distance = 50 ft up + 50 ft down

= 100 ft ✓8

units on parts (a) through (g) ✓9

(h) Do you think Pooh can see Roo's House from the top of the tree? Include units.

Yes, of course he can houses/climb

(This question is not graded)