Name KEY Date ________________ Famous

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AP Calculus TAKE HOME TEST: 4.1-4.5 NO CALCULATOR

Part I: SHORT ANSWER (ALL WORK MUST BE SHOWN FOR CREDIT. ANY CORRECT ANSWER IN THE ABSENCE OF SUPPORTING WORK WILL BE COUNTED INCORRECT! **GIVE SIMPLIFIED, EXACT ANSWERS!**)

1. If $f(x)=\left(3x^2-4x-1\right)\tan x$, then $f'(0)=$

$$\boxed{f'(x)=(6x-4)(\tan x)+(3x^2-4x-1)(\sec^2 x)}$$

$$f'(0)=(-4)(\tan 0)+(-1)(\sec 0)^2$$

$$f'(0)=0-1^2$$

$$\boxed{f'(0)=-1}$$

2. If $f(x)=3x^{1/3}(2x+1)$, find the values of x for which f is differentiable, that is, find the domain of $f'(x)$. Be sure to show your computation of $f'(x)$ and analysis.

$$f'(x)=x^{-2/3}(2x+1)+3x^{1/3}(2)$$

$$f'(x)=\frac{2x+1}{\sqrt[3]{x^2}}+6\sqrt[3]{x}$$

$D_{f'}: \{x \mid x\neq 0\}$, $\boxed{f \text{ is differentiable } \forall x\neq 0}$

3. If $f(x)=e+\pi x$, then $f'\left(\sqrt{2}\right)=$

$$f'(x)=\pi$$

$$\boxed{f'(\sqrt{2})=\pi}$$

4. The following limit gives $f'(c)$ for some function $f(x)$ at some $x=c$. Identify $f(x)$, $x=c$, then find $f'(x)$, and finally $f'(c)$.

$$\lim_{h\to 0}\frac{3\csc\left(\frac{\pi}{2}+h\right)-3}{h}=$$

$$f(x)=3\csc x$$

$$f'(x)=-3\csc x\cot x$$

$$f'\left(\tfrac{\pi}{2}\right)=-3\left(\csc\tfrac{\pi}{2}\right)\left(\cot\tfrac{\pi}{2}\right)$$

$$f'\left(\tfrac{\pi}{2}\right)=-3(1)(0)$$

$$\boxed{f'\left(\tfrac{\pi}{2}\right)=0}$$

5. If $f(x)=\sqrt[3]{3x}$, then $f'(\sqrt{3})=$

$$f(x)=\sqrt[3]{3}\,x^{1/3}$$
$$f'(x)=\frac{\sqrt[3]{3}}{3}x^{-2/3}$$
$$f'(x)=\frac{\sqrt[3]{3}}{3}\cdot\frac{1}{\sqrt[3]{x^2}}$$

$$f'(\sqrt{3})=\frac{\sqrt[3]{3}}{3}\cdot\frac{1}{\sqrt[3]{\sqrt{3}^2}}$$
$$f'(\sqrt{3})=\frac{\cancel{\sqrt[3]{3}}}{3}\cdot\frac{1}{\cancel{\sqrt[3]{3}}}$$
$$\boxed{f'(\sqrt{3})=\frac{1}{3}}$$

6. Let $f(x)=\begin{cases} cx+d, & x\le 2 \\ x^2-cx, & x>2 \end{cases}$, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

$$\lim_{x\to 2^-} f(x)=f(2)=2c+d$$
$$\lim_{x\to 2^+} f(x)=4-2c$$
$$\text{so } 2c+d=4-2c$$

$$f'(x)=\begin{cases} c, & x\le 2 \\ 2x-c, & x>2 \end{cases}$$
$$\lim_{x\to 2^-} f'(x)=c$$
$$\lim_{x\to 2^+} f'(x)=4-c$$
$$\text{so } c=4-c$$
$$2c=4$$
$$\boxed{c=2}$$

$$\text{so } 2(2)+d=4-2(2)$$
$$4+d=0$$
$$\boxed{d=-4}$$

$$\text{so } c+d=2-4$$
$$\boxed{c+d=-2}$$

7. A particle moves along the x-axis so that at time $t\ge 0$ its position is given by $x(t)=2t^3-21t^2+72t-53$. At what time t is the particle at rest?

$$x'(t)=v(t)=6t^2-42t+72=0$$
$$6(t^2-7t+12)=0$$
$$6(t-4)(t-3)=0$$
$$\boxed{t=4\text{ sec},\ t=3\text{ sec}}$$

8. If $f(x)=(2x-1)\left(\dfrac{x^2-2}{5x-7}\right)$, then $f'(0)=$

$$f'(x)=2\left(\frac{x^2-2}{5x-7}\right)+(2x-1)\left(\frac{(5x-7)(2x)-(x^2-2)(5)}{(5x-7)^2}\right)$$
$$f'(0)=2\left(\frac{2}{7}\right)+(-1)\left(\frac{-5(-2)}{49}\right)$$
$$f'(0)=\frac{4}{7}-\frac{10}{49}$$
$$\boxed{f'(0)=\frac{18}{49}}$$

Part II: FREE RESPONSE (SHOW ALL SET-UPS. INCLUDE UNITS IN ALL ANSWERS. NOTATION, NOTATION, NOTATION. WORK ALL QUESTIONS IN THE SPACE BELOW EACH QUESTION.)

9. A particle moves along a vertical number line and has a position equation for $t \geq 0$ of $y(t) = (3t-1)(t-3)$ with $y(t)$ measured in feet and t measured in seconds.

(a) What is the initial position of the particle?

$y(t) = (3t-1)(t-3) = 3t^2 - 10t + 3$

$\boxed{y(0) = 3\text{ ft}}$

(b) When is the first time the particle is at $y = 0$ on the number line?

$y(t) = (3t-1)(t-3) = 0$

$t = \frac{1}{3}, t = 3$

So $\boxed{t = \frac{1}{3}\text{ sec}}$

(c) What is the particle's displacement on the interval from $t = 0$ to $t = 1$ seconds? Explain what this answer means in terms of the particle's starting position.

$y(t) = (3t-1)(t-3) = 3t^2 - 10t + 3$

Displacement $= y(1) - y(0) = -4 - 3 = \boxed{-7\text{ ft}}$

From $t=0$ to $t=1$ seconds, the particle ended up 7 ft BELOW where he started.

(d) What is the particle's average velocity on the interval from $t = 0$ to $t = 1$ seconds?

$y(t) = (3t-1)(t-3) = 3t^2 - 10t + 3$

$$\text{Avg velocity} = \frac{y(1) - y(0)}{1 - 0} = \frac{-7}{1} = \boxed{-7\text{ ft/sec}}$$

(e) What is the particle's velocity $t = 1$ seconds? Explain what this means in terms of the direction and speed of the particle.

$$y(t) = (3t-1)(t-3) = 3t^2 - 10t + 3$$

$$y'(t) = v(t) = 3(t-3) + (3t-1)(1) = 6t - 10$$

$$\boxed{y'(1) = -4 \text{ ft/sec}}$$

At $t = 1$ sec, the particle is Moving DOWN at 4ft per second.

(f) What is the particle's acceleration at $t = 1$ seconds? Explain what this means in terms of the velocity of the particle.

$$y''(t) = v'(t) = a(t) = 6$$

$$\boxed{a(1) = 6 \text{ ft/sec}^2}$$

At $t=1$ seconds the particle's velocity is INCREASING by 6 ft/sec per second.

(g) At what time does the particle change direction? Justify.

$$y'(t) = v(t) = 6t - 10 = 0$$

$$t = \frac{10}{6}$$

$$t = \frac{5}{3} \text{ seconds}$$

The particle changes directions at $t = \frac{5}{3}$ seconds since $v(t)$ changes from negative to positive at $t = \frac{5}{3}$ seconds

(h) At $t = 1$ seconds, is the speed of the particle increasing or decreasing? Justify.

Decreasing since $v(1) < 0$ and $a(1) > 0$