Name KEY Date Thur 10/18/18 Defunct Political Party The Know-Nothings

AP Calculus TEST: 2.1-2.4 , NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

B 1. In the *xy*-plane, the line $5x + y = k$, where k is a constant, is tangent to the graph of $y = 2x^2 + 3x - 1$. What is the value of k?

$y = -5x + k$ $y' = -5$ $y' = 4x + 3$

(A) −2 (B) −9 (C) −5 (D) 7 (E) 4

y-values
$-5x + k = 2x^2 + 3x - 1$
$k = 2x^2 + 8x - 1$

slopes
$-5 = 4x + 3$
$4x = -8$
$x = -2$

So, $k = 2(-2)^2 + 8(-2) - 1$
$= 8 - 16 - 1$
$= -8 - 1$
$= -9$

A 2. If $f(x) = \dfrac{6x+1}{7x-3}$, find $f'(-1)$.

(A) $-\dfrac{1}{4}$ (B) $-\dfrac{11}{100}$ (C) $-\dfrac{25}{16}$ (D) $\dfrac{25}{16}$ (E) $\dfrac{11}{100}$

Quotient Rule
(HodHi Rule)

$f'(x) = \dfrac{(7x-3)(6) - (6x+1)(7)}{(7x-3)^2}$

$f'(-1) = \dfrac{(-10)(6) - (-5)(7)}{(-10)^2}$

$= \dfrac{-60 + 35}{100}$

$= \dfrac{-25}{100}$

$= -\dfrac{1}{4}$

C 3. $\lim\limits_{h \to 0} \dfrac{2(2+h)^5 - 64}{h} =$ $f'(2)$, for $f(x) = 2x^5$

(A) DNE (B) 64 (C) 160 (D) 100 (E) 36

$f'(x) = 10x^4$
$f'(2) = 10(2^4)$
$= 10(16)$
$= 160$

D 4. If $f(x) = \begin{cases} 2ax^2 + x + 2, & x < -1 \\ bx + 3, & x \geq -1 \end{cases}$, what is the value of b that makes $f(x)$ differentiable at $x = -1$?

(A) −1 (B) 1 (C) −3 (D) 3 (E) $-\dfrac{1}{2}$

continuity
$\lim\limits_{x \to -1^-} f(x) = 2a - 1 + 2$
$= 2a + 1$
$f(-1) = -b + 3$
$\lim\limits_{x \to -1^+} f(x) = -b + 3$
So, $2a + 1 = -b + 3$
$b = 2 - 2a$

$f'(x) = \begin{cases} 4ax + 1, & x < -1 \\ b, & x > -1 \end{cases}$

slopes
$\lim\limits_{x \to -1^-} f'(x) = -4a + 1$
$\lim\limits_{x \to -1^+} f'(x) = b$
So, $b = -4a + 1$

So, $2 - 2a = -4a + 1$
$2a = -1$
$\boxed{a = -\frac{1}{2}}$
and $b = 2 - 2(-\frac{1}{2})$ plug back in
$b = 2 + 1$
$\boxed{b = 3}$

E 5. If $f(x) = -2x^3 + \dfrac{3}{x^2} - \sqrt{x} + \dfrac{2}{\sqrt[3]{x^2}}$, then $f'(1) =$

(A) $-\dfrac{11}{6}$ (B) $\dfrac{67}{15}$ (C) $\dfrac{-67}{6}$ (D) $\dfrac{83}{6}$ (E) $\dfrac{-83}{6}$

$f(x) = -2x^3 + 3x^{-2} - x^{1/2} + 2x^{-2/3}$

$f'(x) = -6x^2 - 6x^{-3} - \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-5/3}$

$f'(x) = -6x^2 - \dfrac{6}{x^3} - \dfrac{1}{2\sqrt{x}} - \dfrac{4}{3\sqrt[3]{x^5}}$

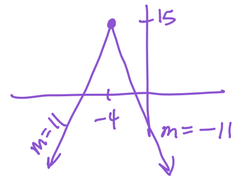
$f'(1) = -6 - 6 - \frac{1}{2} - \frac{4}{3}$
$= -12 - \frac{11}{6} = -\frac{72 - 11}{6} = \frac{-83}{6}$

A 6. If $f(x) = 15 - |11x + 44|$ for all x, what is the value of $f'(4)$

(A) -11 (B) 11 (C) 44 (D) -44 (E) DNE

$f(x) = -|11(x+4)| + 15$

$f(x) = -11|x+4| + 15$



$f'(4) = -11$

Since $x=4$ is to the right of $x=-4$, the x-coordinate of the vertex

E 7. $\frac{d}{dx}\left[(3x^4)(\cos x)\right] =$

(A) $-12x^3 \sin x$ (B) $3x^3(4\cos x + x\sin x)$ (C) $3x^3(4\sin x - x\cos x)$

(D) $12x^3 \sin x$ (E) $3x^3(4\cos x - x\sin x)$

Product Rule

$(12x^3)(\cos x) + (3x^4)(-\sin x)$

$12x^3\cos x - 3x^4 \sin x$

$3x^3(4\cos x - x\sin x)$

$$g(x) = \begin{cases} 4x+1, & x \le 2 \\ x^2+6, & x > 2 \end{cases}$$

$\lim_{x\to 2^-} g(x) = 4(2)+1 = 9$

$\lim_{x\to 2^+} g(x) = (2)^2 + 6 = 10$

A 8. Let f be the function given above. Which of the following statements are true about g?

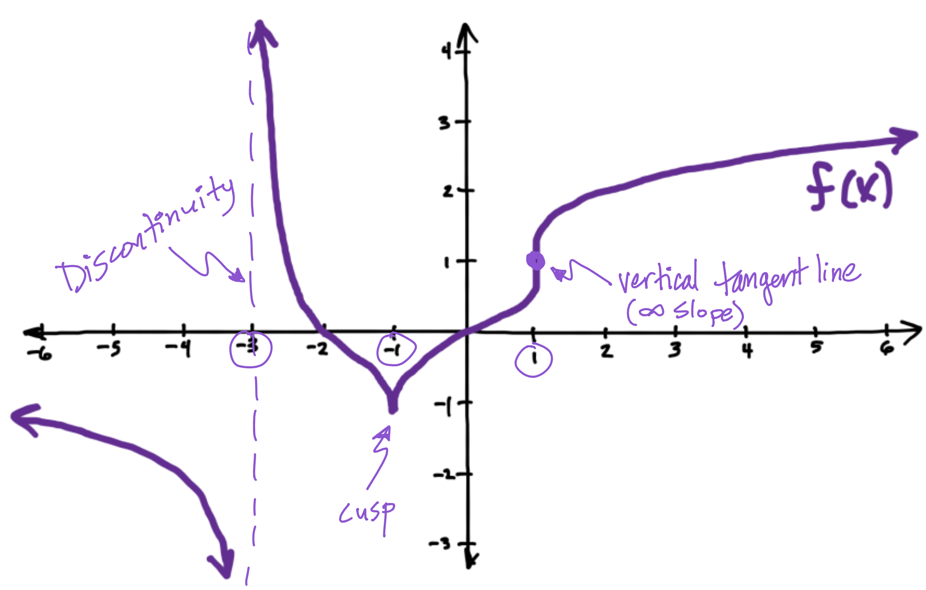
I. $\lim_{x\to 2} g(x)$ exists

II. g is continuous at $x = 2$

III. g is differentiable at $x = 2$

$9 \neq 10$, so $\lim_{x\to 2} g(x) = \text{DNE}$, and so $g(x)$ is not continuous at $x=2$, and so $g(x)$ is not differentiable at $x=2$.

(A) None (B) I only (C) III only (D) I & II only (E) I, II, & III



A 9. The graph of a function $f(x)$ is given above. The graph of $f(x)$ has a vertical asymptote at $x = -3$, a vertical tangent line at $x = 1$, and x-intercepts at $x = -2$ and $x = 0$. For what values of x is the function $f(x)$ is **not** differentiable?

(A) $-3,\ -1,\ 1$ only (B) $-3,\ -1$ only (C) $-3,\ 1$ only (D) -3 only (E) $-1,\ 1$ only

Part II: Free Response—Do all work in the space provided.

10. If $g(x) = 2x^3 - 4x^2 + 3x - 11$

(a) Let $P(x) = g'(x)$. Find $P(x)$ and $P'(x)$.

$P(x) = g'(x) = 6x^2 - 8x + 3$ ✓1

$P'(x) = g''(x) = 12x - 8$ ✓2

(b) Find $P(-1)$ and $P'(-1)$.

$P(-1) = 6(-1)^2 - 8(-1) + 3$
$= 6 + 8 + 3$
$= 14 + 3$
$= 17$ ✓3

$P'(-1) = 12(-1) - 8$
$= -12 - 8$
$= -20$ ✓4

(c) Find the equation of the tangent line, in Taylor Form, of $P(x)$ at $x = -1$.

pt: $(x_1, y_1) = (-1, 17)$

$m = P'(-1) = -20$

$y = y_1 + m(x - x_1)$

$\boxed{y = 17 - 20(x+1)}$

✓5 for slope

✓6 for equation

(d) Find the equation of the normal line, in Taylor Form, of $P(x)$ at $x = -1$.

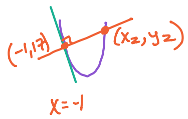
pt: $(x, y) = (-1, 17)$

$m_\perp = \frac{-1}{P'(-1)} = \frac{1}{20}$

$y = y_1 + m(x - x_1)$

$\boxed{y = 17 + \frac{1}{20}(x+1)}$ ✓7

(e) The equation of the normal line to $P(x)$ at $x=-1$ intersects the graph of $P(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.



$$17+\frac{1}{20}(x+1)=6x^2-8x+3$$ √8 set up equation

$$20\cdot\left[17+\frac{1}{20}(x+1)\right]=\left[6x^2-8x+3\right]\cdot 20$$

$$340+x+1=120x^2-160x+60$$

$$0=120x^2-161x-281$$

$$\begin{array}{r} -341 \\ +60 \\ \hline -281 \end{array}$$

$$(x+1)(120x-281)=0$$

Since $x=-1$ is a Solution

force this factor, then check middle term: $120x-281x=-161x$

so, $x=-1$ & $\boxed{x=\frac{281}{120}}$ √9