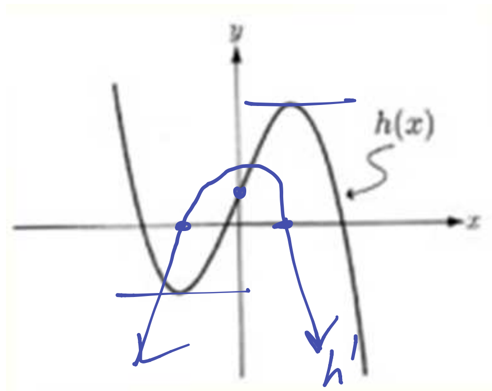
Name KEY Date 10/18/2017 Pencil Brand Mirado

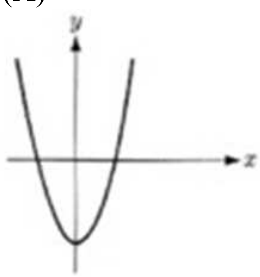
AP Calculus TEST: 2.1-2.4 , NO CALCULATOR 18 pts. (Black Warrior!)

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

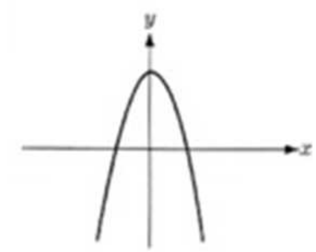


B 1. The graph of a function h is shown above. Which of the following could be the graph of h', the derivative of h?

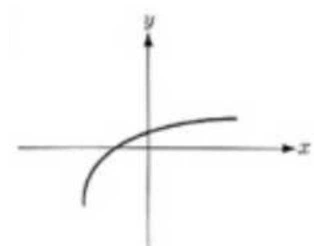
(A)



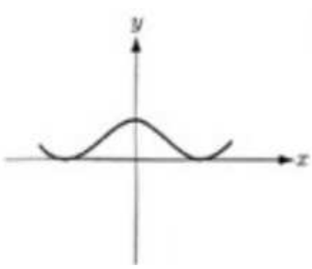
(B)



(C)



(D)



(E)

D 2. In the *xy*-plane, the line $2x - y = k$, where k is a constant, is tangent to the graph of $y = \frac{3}{2}x^2 - 4x + 1$. What is the value of k?

(A) 2 (B) -2 (C) 3 (D) 5 (E) -1

$y = 2x - k$ $y = \frac{3}{2}x^2 - 4x + 1$; $y' = 2$, $y' = 3x - 4$

$y = y$ $y' = y'$

$2x - k = \frac{3}{2}x^2 - 4x + 1$ $2 = 3x - 4$

So, $2(2) - k = \frac{3}{2}(2^2) - 4(2) + 1$ $6 = 3x$

$4 - k = 6 - 8 + 1$ $x = 2$

$4 - k = -1$

$\boxed{5 = k}$

$$f(x)=\begin{cases}2cx+d & \text{for } x\le -1\\ x^2+cx & \text{for } x>-1\end{cases}$$

B 3. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=-1$, what is the value of $c\cdot d$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

Continuity

$-2c+d=1-c$

so, $-2(-2)+d=1-(-2)$

$4+d=3$

$\boxed{d=-1}$

so, $c\cdot d=(-2)(-1)$

$=2$

$f'(x)=\begin{cases}2c & ,x<-1\\ 2x+c & ,x>-1\end{cases}$

slopes

$2c=-2+c$

$\boxed{c=-2}$

D 4. If $y=\dfrac{3x+2}{2x+3}$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{13}{(2x+3)^2}$ (B) $\dfrac{-13}{(2x+3)^2}$ (C) $\dfrac{-1}{(2x+3)^2}$ (D) $\dfrac{5}{(2x+3)^2}$ (E) $\dfrac{-5}{(2x+3)^2}$

$\dfrac{dy}{dx}=\dfrac{(2x+3)(3)-(3x+2)(2)}{(2x+3)^2}$

$\dfrac{dy}{dx}=\dfrac{6x+9-6x-4}{(2x+3)^2}$

$\dfrac{dy}{dx}=\dfrac{5}{(2x+3)^2}$

A 5. $\lim\limits_{h\to 0}\dfrac{2\cos\left(\cancel{\frac{7\pi}{6}+h}\right)-2\cos\left(\frac{7\pi}{6}\right)}{h}=f'\left(\frac{7\pi}{6}\right)$

(A) 1 (B) -1 (C) $\sqrt{3}$ (D) $-\sqrt{3}$ (E) $-\sqrt{2}$

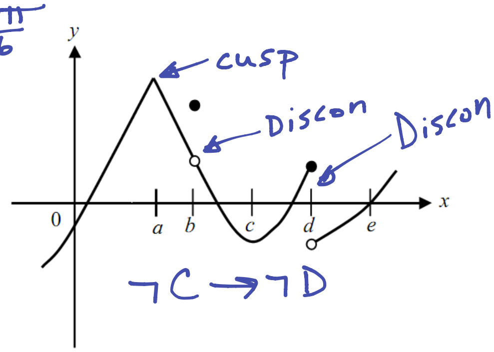
$f(x)=2\cos x,\ c=\frac{7\pi}{6}$

$f'(x)=-2\sin x$

$f'\left(\frac{7\pi}{6}\right)=-2\sin\frac{7\pi}{6}$

$=(-2)\left(-\frac{1}{2}\right)$

$=1$



Graph of f

D 6. The graph of a function f is shown above. At which value(s) of x is f not differentiable?

I. $x=a$ II. $x=b$ III. $x=d$

(A) I only (B) I & II only (B) II & III only (C) I & III only (D) I, II, & III

$\lim_{x \to 3^-} g(x) = 5$

$\lim_{x \to 3^+} g(x) = 5$

$g(3) = 5$

$$g(x) = \begin{cases} x+2, & x \le 3 \\ x^2 - 4, & x > 3 \end{cases}$$

D 7. Let f be the function given above. Which of the following statements are true about g ?

I. $\lim_{x \to 3} g(x)$ exists ✓ $= 5$

II. g is continuous at $x = 3$ ✓ $5 = 5 = 5$

III. g is differentiable at $x = 3$ ✗

$g'(x) = \begin{cases} 1, & x < 3 \\ 2x, & x > 3 \end{cases}$

$\lim_{x \to 3^-} g'(x) = 1$

$\lim_{x \to 3^+} g'(x) = 6$

$1 \neq 6$

So, g is not diff'able at $x = 3$

(A) None (B) I only (C) II only (D) I & II only (E) I, II, & III

B 8. If $f(x) = (x-2)\sin x$, then $f'(0) =$

product Rule

(A) -3 (B) -2 (C) 0 (D) 2 (E) 3

$f'(x) = (1)(\sin x) + (x-2)\cos x$

$f'(0) = \sin 0 + (-2)\cos 0$

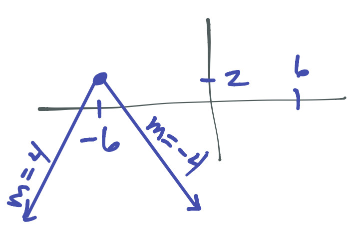
$= 0 - 2(1)$

$= -2$

A 9. If $f(x) = 2 - 4|x+6|$ for all x, then the value of the derivative $f'(x)$ at $x = 6$ is

(A) -4 (B) 0 (C) 4 (D) 2 (E) DNE

$f(x) = -4|x+6| + 2$



$f'(6) = -4$

(to right of vertex)

Part II: Free Response—Do all work in the space provided.

12. If $g(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + 5$

(a) Let $P(x) = g'(x)$. Find $P(x)$ and $P'(x)$.

$P(x) = 2x^2 + x - 1$ √1

$P'(x) = 4x + 1$ √2

(b) Find $P(1)$ and $P'(1)$.

$P(1) = 2 + 1 - 1 = 2$ √3

$P'(1) = 4 + 1 = 5$ √4

(c) Find the equation of the <u>tangent</u> line, in Taylor Form, of $P(x)$ at $x = 1$.

pt: $(1,2)$

$m = 5$

eq: $y = 2 + 5(x-1)$ √5 √6

(d) Find the equation of the <u>normal</u> line, in Taylor Form, of $P(x)$ at $x = 1$.

pt: $(1,2)$

$m = -\frac{1}{5}$ (opp recip)

eq: $y = 2 - \frac{1}{5}(x-1)$ √7

(e) The equation of the normal line to $P(x)$ at $x = 1$ intersects the graph of $P(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.

$2 - \frac{1}{5}(x-1) = 2x^2 + x - 1$ √8

$5\left[2 - \frac{1}{5}(x-1)\right] = 5\left[2x^2 + x - 1\right]$

$10 - (x-1) = 10x^2 + 5x - 5$

$10 - x + 1 = 10x^2 + 5x - 5$

$0 = 10x^2 + 6x - 16$

$0 = (x-1)(10x + 16)$

So, $x = 1$ & ... $x = -\frac{16}{10} = -\frac{8}{5}$ √9