Name KEY Date ______ Common Injury punctured Aorta

AP Calculus TEST: 2.1-2.4 , NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

D 1. If $y = \sec x$, then $\frac{d^2y}{dx^2} =$

(A) $\sec^3 x \tan x$ (B) $\sec x \tan x$ (C) $\sec x\left(2\sec^2 x + 1\right)$ (D) $\sec x\left(2\sec^2 x - 1\right)$

$\frac{dy}{dx} = \sec x \tan x$

$\frac{d^2y}{dx^2} = (\sec x \tan x)\tan x + (\sec x)(\sec^2 x)$ *product rule

$= \sec x \cdot \tan^2 x + \sec^3 x$

$= \sec x(\tan^2 x + \sec^2 x)$

$= \sec x(\sec^2 x - 1 + \sec^2 x)$ *P.I.D.

$= \sec x(2\sec^2 x - 1)$

A 2. If $g(x) = \frac{x+2}{x-2}$, then $g'(-2) =$

(A) $-\frac{1}{4}$ (B) -1 (C) 1 (D) $\frac{1}{4}$

$g'(x) = \frac{(x-2)(1) - (x+2)(1)}{(x-2)^2}$

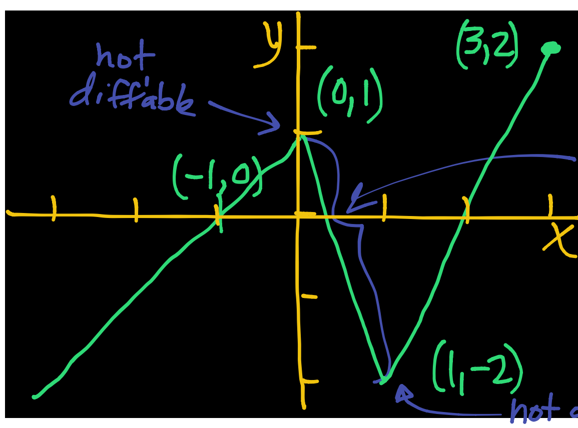
$g'(x) = \frac{x-2-x-2}{(x-2)^2}$

$g'(x) = \frac{-4}{(x-2)^2}$

$g'(-2) = \frac{-4}{(-4)^2}$

$= \frac{-4}{16}$

$= -\frac{1}{4}$



C 3. The function $K(x)$, whose graph is composed of straight line segments is shown above. Which of the following is true for $K(x)$ on the open interval $(-2,3)$?

I. $\lim_{x\to 0} K(x)$ exists ✓ $= 1$

II. $K(x)$ is differentiable for all $x \in (-2,3)$ ✗ not at $x = 0, 1$

III. The derivative of $K(x)$ is positive on the interval $(1,3)$ ✓ only dec for $x \in (0,1)$

(A) I only (B) II only (C) I and III only (D) I, II, and III

A 4. If $f(x) = -x^5 + \frac{1}{x} - \sqrt[3]{x} + \frac{1}{\sqrt{x^5}}$, then $f'(1) =$

(A) $-\frac{53}{6}$ (B) $-\frac{58}{15}$ (C) $\frac{58}{15}$ (D) $\frac{53}{6}$

$f(x) = -x^5 + x^{-1} - x^{1/3} + x^{-5/2}$

$f'(x) = -5x^4 - x^{-2} - \frac{1}{3}x^{-2/3} - \frac{5}{2}x^{-7/2}$

$f'(x) = -5x^4 - \frac{1}{x^2} - \frac{1}{3\sqrt[3]{x^2}} - \frac{5}{2\sqrt{x^7}}$

$f'(1) = -5 - 1 - \frac{1}{3} - \frac{5}{2}$

$= -6 - \frac{17}{6}$

$= -\frac{53}{6}$

$\text{So } \frac{7}{4}x-\frac{3}{4}=x^3+x+c \;\&\; \frac{7}{4}=3x^2+1$

$3x^2=\frac{3}{4}$

$-4y=-7x+3,\ y=\frac{7}{4}x-\frac{3}{4},$ $\quad x^2=\frac{1}{4},\ x=\pm\frac{1}{2}$

A 5. If the line $7x-4y=3$ is tangent in the first quadrant to the curve $y=x^3+x+c$, then $c=$ in 1st Quad, $x=\frac{1}{2}$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ $y'=\frac{7}{4},\ y'=3x^2+1$

$\text{So, } \frac{7}{4}\left(\frac{1}{2}\right)-\frac{3}{4}=\frac{1}{8}+\frac{1}{2}+c,\ c=-\frac{1}{2}$

_____ 6. The function $f(x)=x^4+3x^3+2x+4$ must have a zero/root between which of the following values of x? IVT

(A) -2 and 1 (B) 1 and 2 (C) 2 and 3 (D) 3 and 4

$f(-2)=16-24-4+4$
$=-8<0$
$f(1)=1+3+2+4$
$=10>0$

$$g(x)=\begin{cases} x+2, & x\le 3 \\ 4x-7, & x>3 \end{cases}$$

$\lim_{x\to3^-} g(x)=g(3)=5$

$\lim_{x\to3^+} g(x)=5$

D 7. Let f be the function given above. Which of the following statements are true about g?

I. $\lim_{x\to3} g(x)$ exists ✓ $=5$

II. g is continuous at $x=3$ ✓ $5=5=5$

III. g is differentiable at $x=3$ ✗ $4\ne1$

$g'=\begin{cases} 1, & x<3 \\ 4, & x>3 \end{cases}$

$1\ne4$, not diff'able

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

D 8. What are all the horizontal asymptotes for the graph of $f(x)=\frac{5x}{\sqrt{x^2+1}}$? $\approx \frac{5x^1}{1x^1}$

(A) $y=0$ only (B) $y=5$ only (C) $y=-5$ only (D) $y=5$ and $y=-5$

$\lim_{x\to\infty} f(x)=5$ (plug in pos & neg into $f(x)$ to get sign of ∞)

$\lim_{x\to-\infty} f(x)=-5$

D 9. $\lim_{h\to0}\frac{9\left(\frac{1}{3}(\cancel{x+h})\right)^3-9\left(\frac{1}{3}x\right)^3}{h}=f'(x)$

(A) $\frac{x^2}{3}$ (B) 0 (C) $9x^2$ (D) x^2

$f(x)=9\left(\frac{1}{3}x\right)^3$
$=(9)\left(\frac{1}{27}x^3\right)$
$=\frac{1}{3}x^3$
$f'(x)=x^2$

Part II: Free Response—Do all work in the space provided. Show all steps. Use proper notation.

10. If $f(x) = \frac{5}{3}x^3 + 2x^2 - 3x + 11$

(a) Let $Q(x) = f'(x)$. Find $Q(x)$ and $Q'(x)$.

$Q(x) = 5x^2 + 4x - 3$ √1

$Q'(x) = 10x + 4$ √2

(b) Find $\lim_{x\to\infty} \frac{Q(x)}{[Q'(x)]^2} =$

$\lim_{x\to\infty} \frac{5x^2+4x-3}{[10x+4]^2}$

$\lim_{x\to\infty} \frac{5x^2+4x-3}{100x^2+80x+16} = \frac{5}{100} = \frac{1}{20}$ √3

(c) Find $Q(-2)$ and $Q'(-2)$.

$Q(-2) = 5(4) - 8 - 3 = 9$ √4

$Q'(-2) = -20 + 4 = -16$ √5

(d) Find the equation of the tangent line, in Taylor Form, of $Q(x)$ at $x = -2$.

pt: $(-2, 9)$

m: -16

eq: $y = 9 - 16(x+2)$ √6

(e) Find the equation of the normal line, in Taylor Form, of $Q(x)$ at $x = -2$.

$\hookrightarrow$ perp. to tan. line

pt: $(-2, 9)$

m: $\frac{1}{16}$ (opp recip)

eq: $y = 9 + \frac{1}{16}(x+2)$ ✓7

(f) The equation of the normal line to $Q(x)$ at $x = -2$ intersects the graph of $Q(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.

$$9 + \frac{1}{16}(x+2) = 5x^2 + 4x - 3$$ ✓8

$$16\left[9 + \frac{1}{16}(x+2)\right] = 16\left[5x^2 + 4x - 3\right]$$

$$144 + x + 2 = 80x^2 + 64x - 48$$

$$0 = 80x^2 + 63x - 194$$

$$0 = (x+2)(80x - 97)$$

so, $x = -2$ (duh!) & ... $x = \frac{97}{80}$ ✓9