Name KEY Date________ Famous General________

AP Calculus TEST: 2.1-2.4 , NO CALCULATOR

Part Ein: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

A 1. In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph for $f(x)=x^2+3x+1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

$y=-x+k$
$y'=-1$

$f(x)=x^2+3x+1$
$f'(x)=2x+3$

y-values
$-x+k=x^2+3x+1$
$2+k=4-6+1$
$\boxed{k=-3}$

Slopes
$-1=2x+3$
$-4=2x$
$\boxed{x=-2}$

$$g(x)=\begin{cases} ax^2+bx+2, & \text{for } x\le 1 \\ \dfrac{2b}{x}-a & \text{for } x>1 \end{cases}$$

A 2. Let g be the function defined above, where a and b are constants. If g is differentiable at $x=1$, what is the value of a?

(A) $-\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $-\frac{1}{2}$ (E) No such value exists

y-values
$a+b+2=2b-a$
$\boxed{2a+2=b}$

$g'(x)=\begin{cases} 2ax+b, & x<1 \\ -\dfrac{2b}{x^2}, & x>1 \end{cases}$

Slopes
$2a+b=-2b$
$2a=-3b$
$\boxed{b=-\frac{2}{3}a}$

So
$2a+2=-\frac{2}{3}a$
$2a+\frac{2}{3}a=-2$
$\frac{8}{3}a=-2$
$a=-\frac{2}{1}\cdot\frac{3}{8}$
$\boxed{a=-\frac{3}{4}}$

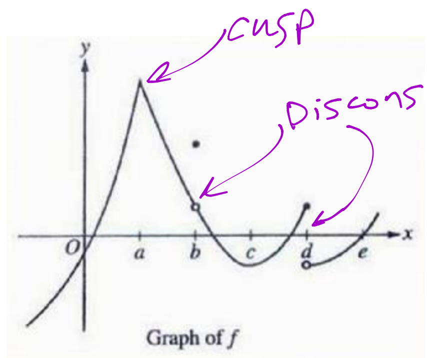
D 3. If $y=\dfrac{3x-4}{5x+7}$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{30x-1}{(5x+7)^2}$ (B) $\dfrac{2x+3}{(5x+7)^2}$ (C) $-\dfrac{41}{(5x+7)^2}$ (D) $\dfrac{41}{(5x+7)^2}$ (E) $-\dfrac{1}{(5x+7)^2}$

$\dfrac{dy}{dx}=\dfrac{(5x+7)(3)-(3x-4)(5)}{(5x+7)^2}=\dfrac{15x+21-15x+20}{(5x+7)^2}=\dfrac{41}{(5x+7)^2}$

B 4. $\displaystyle\lim_{h\to 0}\frac{4\cos\left(\frac{3\pi}{2}+h\right)-4\cos\frac{3\pi}{2}}{h}=$ (A) -4 (B) 4 (C) 0 (D) -1 (E) DNE

$f(x)=4\cos x$
$f'(x)=-4\sin x$
$f'\left(\frac{3\pi}{2}\right)=-4\sin\frac{3\pi}{2}$
$=(-4)(-1)$
$=4$



Graph of f

E ____ 5. The graph of a function f is shown above. At which value(s) of x is f not differentiable?

(A) a (B) a and b (C) a and d (D) b and d (E) a, b, and d

$$h(x)=\begin{cases}4x-3, & x\le 2\\ \frac{3}{2}x^2-2x+3, & x>2\end{cases}$$

$\lim_{x\to 2^-} h(x) = h(2) = 5$

$\lim_{x\to 2^+} h(x) = 6-4+3 = 5$ → continuous

E ____ 6. Let g be the function given above. Which of the following statements are true about g?

I. $\lim_{x\to 2} h(x)$ exists ✓

II. h is continuous at $x=2$ ✓

III. h is differentiable at $x=2$ ✓

$h'(x)=\begin{cases}4, & x<2\\ 3x-2, & x>2\end{cases}$

$\lim_{x\to 2^-} h'(x) = 4$, $\lim_{x\to 2^+} h'(x) = 4$ → diff'able

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

D ____ 7. Which of the following is the equation of the normal line to the function $f(x)= x^2+3x-5$ at $x=1$?

(A) $5x-y=-4$ (B) $x-5y=-4$ (C) $5x+y=-4$ (D) $x+5y=-4$ (E) $-5x+y=-4$

$f(1)=1+3-5$, $f'(x)=2x+3$

$f(1)=-1$, $f'(1)=5$

pt: $(1,-1)$, $m_N=-\frac{1}{5}$

$y=-1-\frac{1}{5}(x-1)$

$5y=5(-1-\frac{1}{5}(x-1))$

$5y=-5-x+1$

$x+5y=-4$

C ____ 8. If $f(x)=x^2\sin(x)-\sqrt{x^3}$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$f'(x) = 2x\sin x + x^2\cos x - \frac{3}{2}x^{1/2}$

$f'(0) = 0 + 0 - 0$

$= 0$

B ____ 9. If $f(x)=x^3+kx^2+x-3$, and if $f'(-2)=17$, then $k=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$f'(x) = 3x^2 + 2kx + 1$

$f'(-2) = 12 - 4k + 1 = 17$

$-4k = 17-13$

$-4k = 4$

$\boxed{k=-1}$

Part Dos: Free Response—Do all work in the space provided. Show all steps. Use proper notation.

10. If $f(x)=\frac{2}{3}x^3+\frac{3}{2}x^2-x+5$

(a) Let $Q(x)=f'(x)$. Find $Q(x)$ and $Q'(x)$.

$Q(x)=2x^2+3x-1$ ✓1

$Q'(x)=4x+3$ ✓2

(b) Find $\lim_{x\to\infty}\frac{Q'(x)}{Q(x)}=$ $\lim_{x\to\infty}\frac{4x+3}{2x^2+3x-1}=0$ ✓3

(c) Find $Q(-2)$ and $Q'(-2)$.

$Q(-2)=8-6-1=1$ ✓4

$Q'(-2)=-8+3=-5$ ✓5

(d) Find the equation of the <u>tangent</u> line, in Taylor Form, of $Q(x)$ at $x=-2$.

pt: $(-2,1)$, $m=-5$

$\underbrace{y=1-5(x+2)}_{✓6}$

(e) Find the equation of the <u>normal</u> line, in Taylor Form, of $Q(x)$ at $x = -2$.

pt: $(-2, 1)$, $m = \frac{1}{5}$

$$\underbrace{y = 1 + \frac{1}{5}(x+2)}_{\checkmark 7}$$

(f) The equation of the normal line to $Q(x)$ at $x = -2$ intersects the graph of $Q(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.

$$y = \overbrace{1 + \frac{1}{5}(x+2) = 2x^2 + 3x - 1}^{\checkmark 8} = Q(x)$$

$5 + x + 2 = 10x^2 + 15x - 5$ ($\times 5$ on both sides)

$x = -2$ is a soln so $(x+2)$ is a factor $\longrightarrow$

$10x^2 + 14x - 12 = 0$

$(x+2)(10x - 6) = 0$

$x = -2$ or $\boxed{x = \frac{6}{10} = \frac{3}{5}}$ $\checkmark 9$