Name KEY Date___________ Pasta Shape____________

AP Calculus TEST: 2.1-2.3, NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question. Attach any scratch work to the back of this test upon completion.

E 1. In the xy-plane, the line $6x + y = 2$, where k is a constant, is tangent to the graph of $y = 2k + x^2$. What is the value of k? $y = -6x + 2$ $y' = 2x$

(A) -3 (B) 3 (C) $-\frac{2}{11}$ (D) 2 (E) $\frac{11}{2}$

$y' = -6$

$-6x + 2 = 2k + x^2 \rightarrow -6 = 2x$

$x = -3$

so $-6(-3) + 2 = 2k + (-3)^2$

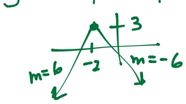
$18 + 2 = 2k + 9$

$2k = 11$

$k = \frac{11}{2}$

E 2. Which of the following is/are true regarding the function $f(x) = 3 - |6x + 12|$?

$y = -6|x+2| + 3$



I. $f'(-2) = DNE$ ✓

II. $f'(0) = 6$ ✗

III. $f(x)$ is continuous for all x ✓

(A) I only (B) III only (C) I and II only (D) I, II, and III (E) I and III only

$$f(x) = \begin{cases} ax^2 + bx + 1 & \text{for } x \le -2 \\ -3ax + 2b & \text{for } x > -2 \end{cases}$$

C 3. Let f be the function defined above, where a and b are constants. If f is differentible at $x = -2$, what is the value of $a \div b$?

(A) -3 (B) $\frac{1}{6}$ (C) 1 (D) 6 (E) No such values exist

y-values

$4a - 2b + 1 = 6a + 2b$

$2b + 1 = 8a$

So, $2b + 1 = 8b$

$1 = 6b$

$b = \frac{1}{6}, a = \frac{1}{6}$

so $a \div b = \frac{1/6}{1/6} = 1$

$f' = \begin{cases} 2ax + b, & x < -2 \\ -3a, & x > -2 \end{cases}$

$-4a + b = -3a$

$b = a$

B 4. If $y = 3x^2(x+2)^2$, then $\frac{dy}{dx} =$

(A) $12x^3 + 18x^2 + 24x$ (B) $12x^3 + 36x^2 + 24x$ (C) $12x^3 + 24x$ (D) $12x^3 + 12x$ (E) $9x^2 + 12x$

$y = 3x^2(x^2 + 4x + 4)$

$y = 3x^4 + 12x^3 + 12x^2$

$\frac{dy}{dx} = 12x^3 + 36x^2 + 24x$

A 5. $\lim_{h \to 0} \frac{2\cos\left(\cancel{\frac{4\pi}{3} + h}\right) - 2\cos\frac{4\pi}{3}}{h} =$ (A) $\sqrt{3}$ (B) 1 (C) $-\sqrt{3}$ (D) -1 (E) $\sqrt{2}$

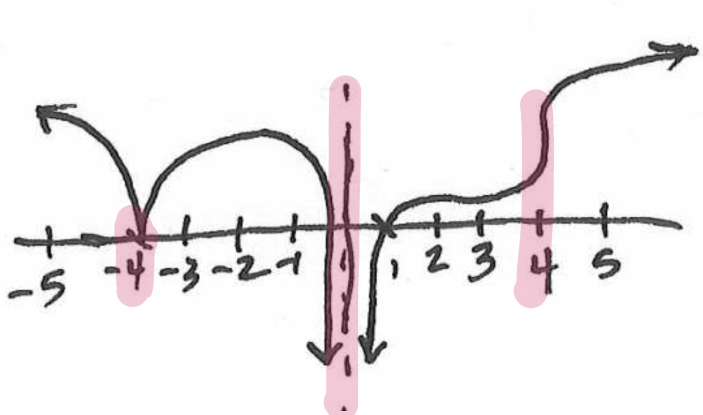
$f(x) = 2\cos x$ $c = \frac{4\pi}{3}$

$f'(x) = -2\sin x$

$f'\left(\frac{4\pi}{3}\right) = -2\sin\frac{4\pi}{3}$

$= -2\left(-\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$



C

6. The graph of a function $f(x)$ is given above. The graph of $f(x)$ has a vertical asymptote at $x = 0$, a vertical tangent line at $x = 4$, and x-intercepts at $x = -4$, $x = -0.5$, and $x = 1$. For what values of x is the function $f(x)$ is **not** differentiable?

   I. $x = -4$
   II. $x = 0$
   III. $x = 3$
   IV. $x = 4$

(A) I & II only (B) I, II, & III only (C) I, II, & IV only (D) I & IV only (E) I, II, III, & IV

$$g(x) = \begin{cases} 6x - 2, & x < -1 \\ -3x^2 - 5, & x \geq -1 \end{cases}$$

$g' = \begin{cases} 6, & x < -1 \\ -6x, & x \geq -1 \end{cases}$ $\lim_{x \to -1^-} g' = 6$, $\lim_{x \to -1^+} g' = 6$. So, g is diff'ble at $x = -1$.

E

7. Let g be the function given above. Which of the following statements are true about g?

   I. $\lim_{x \to -1} g(x)$ exists
   II. g is continuous at $x = -1$
   III. g is differentiable at $x = -1$

$\lim_{x \to -1^-} g(x) = -6 - 2 = -8$
$\lim_{x \to -1^+} g(x) = g(-1) = -3 - 5 = -8$
So, g is continuous at $x = -1$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

$$\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x - 1} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \text{ (Alt. form)}$$

A

8. The above limit represents $f'(c)$, the derivative of some function $f(x)$ at some $x = c$. What are $f(x)$ and $x = c$?

So, $f(x) = \sqrt{x+3}$, $c = 1$

(A) $f(x) = \sqrt{x+3}$, $c = 1$ (B) $f(x) = \sqrt{x+3} - 2$, $c = 1$ (C) $f(x) = \sqrt{x+3}$, $c = 2$
(D) $f(x) = \sqrt{x}$, $c = 3$ (E) $f(x) = \sqrt{x+2}$, $c = 1$

D

9. $\frac{d}{dx}\left[\frac{2x^2 - 3\sqrt[3]{x} + 1}{\sqrt[3]{x}}\right] = \frac{d}{dx}\left[\frac{2x^2}{x^{1/3}} - \frac{3x^{1/3}}{x^{1/3}} + \frac{1}{x^{1/3}}\right] = \frac{d}{dx}\left[2x^{5/3} - 3 + x^{-1/3}\right] = \frac{10}{3}x^{2/3} - \frac{1}{3}x^{-4/3}$

(A) $\frac{-10x^2 - 1}{3\sqrt[3]{x^4}}$ (B) $\frac{10x^2 + 1}{3\sqrt[4]{x^3}}$ (C) $\frac{10x^2 - 1}{3\sqrt[4]{x^3}}$ (D) $\frac{10x^2 - 1}{3\sqrt[3]{x^4}}$ (E) $\frac{10x^2 + 1}{3\sqrt[3]{x^4}}$

$\frac{1}{3}x^{-4/3}\left[10x^2 - 1\right] = \frac{10x^2 - 1}{3\sqrt[3]{x^4}}$

Part II: Free Response—Do all work below in the space provided.

10. If $f(x) = x^3 + 2x^2 + 4x + 3$

(a) Let $P(x) = f'(x)$. Find $P(x)$ and $P'(x)$. $P(x) = f'(x) = 3x^2 + 4x + 4$ $\sqrt{1}$ $P'(x) = f''(x) = 6x + 4$ $\sqrt{2}$
(b) Find $P(1)$ and $P'(1)$. $P(1) = 3 + 4 + 4 = 11$ $\sqrt{3}$ $P'(1) = 6 + 4 = 10$ $\sqrt{4}$
(c) Find the equation of the <u>tangent</u> line, in Taylor Form, of $P(x)$ at $x = 1$. pt: $(1, 11)$ m: 10 eq: $y = 11 + 10(x - 1)$ $\sqrt{5}$

(d) Find the equation of the normal line, in Taylor Form, of $P(x)$ at $x=1$.

pt: (1,11)

$m_N: -\frac{1}{10}$ eq: $y = 11 - \frac{1}{10}(x-1)$ √6 √7

(e) The equation of the normal line to $P(x)$ at $x=1$ intersects the graph of $P(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.

$11 - \frac{1}{10}(x-1) = 3x^2 + 4x + 4$ √8

$110 - x + 1 = 30x^2 + 40x + 40$

$30x^2 + 41x - 71 = 0$

since (1,11) is 1 of the 2 pts, x=1 is a root & (x-1) is a factor

$(x-1)(30x+71) = 0$

$x = 1$ or $x = -\frac{71}{30}$ √9