Name KEY Date 18 pts total Notable Curmudgeon H. L. Mencken

AP Calculus TEST: 2.1-2.3, NO CALCULATOR

Part WON: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question. Attach any scratch work to the back of this test upon completion.

B 1. In the *xy*-plane, the line $2x - y = 1$, where k is a constant, is tangent to the graph of $y = k - x^2$. What is the value of k?

$y = 2x - 1$ $y' = 2$ $y' = -2x$

(A) -3 (B) -2 (C) -1 (D) 0   (E) 1

y-values
① $2x - 1 = k - x^2$

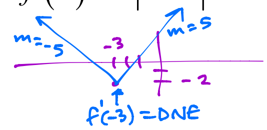
slopes
② $2 = -2x$
$\boxed{x = -1}$
Plug into eq. ①
$2(-1) - 1 = k - (-1)^2$
$-2 - 1 = k - 1$
$-3 = k - 1$
$\boxed{k = -2}$

E 2. Which of the following is/are true regarding the function $f(x) = 5|x+3| - 2$?

I. $f'(3) = \cancel{DNE}$ $f'(-3) = DNE$

II. $f'(-4) = -5$ ✓

III. $f(x)$ is continuous for all x ✓



(A) I only   (B) III only   (C) I and III only   (D) I, II, and III   (E) II and III only

$$f(x) = \begin{cases} ax^2 + bx + 1 & \text{for } x \le -1 \\ -3ax + 2b & \text{for } x > -1 \end{cases}$$

C 3. Let f be the function defined above, where a and b are constants. If f is differentiable at $x = -1$, what is the value of $a + b$?

(A) -2 (B) 5   (C) 0   (D) -3 (E) No such values exist

continuity
$a - b + 1 = 3a + 2b$
① $-2a + 1 = 3b$

$f'(x) = \begin{cases} 2ax + b, & x < -1 \\ -3a, & x > -1 \end{cases}$

Slopes
$-2a + b = -3a$
② $\boxed{a = -b}$
plug into eq. ①
$-2(-b) + 1 = 3b$
$2b + 1 = 3b$
$\boxed{1 = b}$
$\boxed{a = -1}$

So $a + b$
$= -1 + 1 = 0$

A 4. If $y = 2x(x-5)^2$, then $\dfrac{dy}{dx} =$

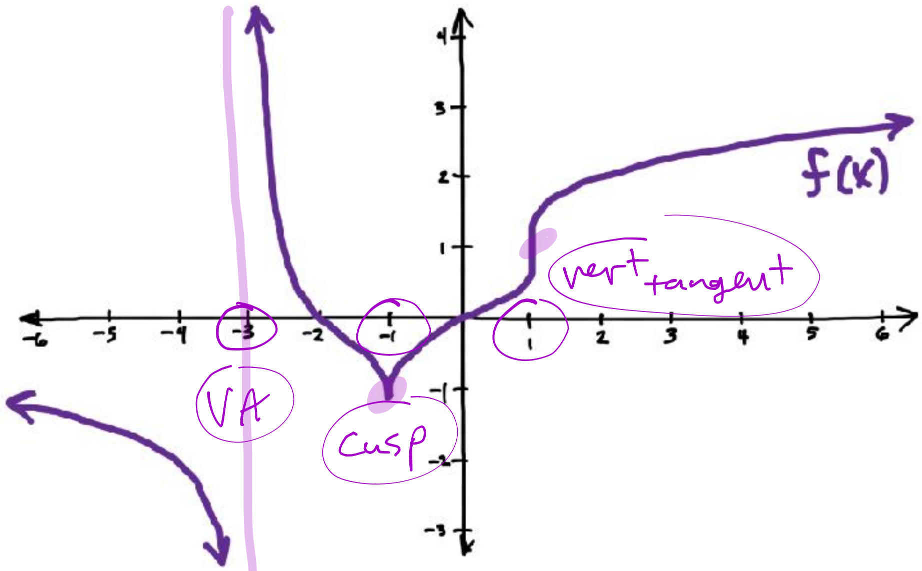
(A) $6x^2 - 40x + 50$ (B) $16x^3 - 120x^2 + 200x$ (C) $6x^2 - 20x + 50$ (D) $4x - 20$ (E) $6x^2 + 50$

$y = 2x(x^2 - 10x + 25)$
$y = 2x^3 - 20x^2 + 50x$
$\frac{dy}{dx} = 6x^2 - 40x + 50$

D 5. $\lim\limits_{h \to 0} \dfrac{6\cos\left(\frac{\pi}{6} + h\right) - 6\cos\frac{\pi}{6}}{h} =$ (A) 0   (B) -6 (C) 6   (D) -3 (E) 3

$= f'\left(\frac{\pi}{6}\right)$ for $f(x) = 6\cos x$

$f'(x) = -6\sin x$
$f'\left(\frac{\pi}{6}\right) = -6\sin\frac{\pi}{6}$
$= -6\left(\frac{1}{2}\right)$
$= -3$



A 6. The graph of a function $f(x)$ is given above. The graph of $f(x)$ has a vertical asymptote at $x=-3$, a vertical tangent line at $x=1$, and x-intercepts at $x=-2$ and $x=0$. For what values of x is the function $f(x)$ is **not** differentiable?

(A) $-3,\ -1,\ 1$ only (B) $-3,\ -1$ only (C) $-3,\ 1$ only (D) -3 only (E) $-1,\ 1$ only

$$g(x)=\begin{cases}7x^2-2, & x<2\\ 26, & x=2\\ 14x-2, & x>2\end{cases}$$

$\lim_{x\to 2^-} g(x) = 26$

$f(2) = 26$

$\lim_{x\to 2^+} g(x) = 26$

D 7. Let g be the function given above. Which of the following statements are true about g?

I. $\lim_{x\to 2} g(x)$ exists ✓

II. g is continuous at $x=2$ ✓

~~III.~~ g is differentiable at $x=2$

$g'(x) = \begin{cases} 14x, & x<2 \\ 14, & x>2 \end{cases}$

$\lim_{x\to 2^-} g'(x) = 28$

$\lim_{x\to 2^+} g'(x) = 14$

$28 \neq 14$ so not diff'able

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

$$\lim_{x\to 0}\frac{(3e^x-x)-3}{x} = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

D 8. The above limit represents $f'(c)$, the derivative of some function $f(x)$ at some $x=c$. What are $f(x)$ and $x=c$?

(A) $f(x)=e^x-x$, $c=3$ (B) $f(x)=3e^x$, $c=0$ (C) $f(x)=3e^x-x-3$, $c=0$

(D) $f(x)=3e^x-x$, $c=0$ (E) $f(x)=3e^x-x$, $c=3$

B 9. $\frac{d}{dx}\left[\frac{3x^3-2\sqrt{x}+1}{\sqrt{x}}\right] = \frac{d}{dx}\left[3x^{5/2}-2+x^{-1/2}\right] = \frac{15}{2}x^{3/2}-\frac{1}{2}x^{-3/2} = \frac{15\sqrt{x^3}}{2}-\frac{1}{2\sqrt{x^3}}$

(A) $\frac{15\sqrt{x^3}}{2}-\frac{\sqrt{x}}{2}$ (B) $\frac{15\sqrt{x^3}}{2}-\frac{1}{2\sqrt{x^3}}$ (C) $\frac{18\sqrt{x^5}-2}{x}$ (D) $3\sqrt{x^5}-2+\frac{1}{\sqrt{x}}$ (E) $18x^2$

Part TOO: Free Response—Do all work below in the space provided.

10. If $f(x) = 5 - 3x - 2x^2 + x^3$

(a) Let $P(x) = f'(x)$. Find $P(x)$ and $P'(x)$. $\boxed{P(x) = f'(x) = -3 - 4x + 3x^2}$ ✓1 $\boxed{P'(x) = -4 + 6x}$ ✓2
(b) Find $P(2)$ and $P'(2)$. $P(2) = -3 - 8 + 12 = 1$ ✓3 $P'(2) = -4 + 12 = 8$ ✓4
(c) Find the equation of the tangent line, in Taylor Form, of $P(x)$ at $x = 2$. pt: $(2, 1)$, $m = 8$ $\boxed{y = 1 + 8(x - 2)}$ ✓5

(d) Find the equation of the normal line, in Taylor Form, of $P(x)$ at $x = 2$.

pt: $(2,1)$, $m_N = -\frac{1}{8}$

$$\boxed{y = 1 - \frac{1}{8}(x-2)}$$ √6

(e) The equation of the normal line to $P(x)$ at $x = 2$ intersects the graph of $P(x)$ at another x-value. Find this x-value. Show the work that leads to your answer.

$$1 - \frac{1}{8}(x-2) = -3 - 4x + 3x^2$$ √7

*multiply both sides by 8

$$8 - (x-2) = -24 - 32x + 24x^2$$

$$8 - x + 2 = -24 - 32x + 24x^2$$

$$24x^2 - 31x - 34 = 0$$ √8

$$(x-2)(24x+17) = 0$$

$x = 2$, $\boxed{x = -\frac{17}{24}}$ √9

original point of tangency

9 F.R. points