Name KEY Date ___ Obscure Constellation ___

BC Calculus TEST: 4.1 - 4.10 (not including LOG DIFF), NO CALCULATOR

Part Eins: Vielen choices—Put the correct CAPITAL letter in the space to the left of each question.

A 1. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

$\frac{4}{1+16x^2}$ → $\frac{4}{1+1}=2$

A 2. In the *xy*-plane, the line $x+y=k$, where *k* is a constant, is tangent to the graph of $y=x^2+3x+1$. What is the value of *k*? $-2-1=k=-3$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

$2x+3=-1$, $2x=-4$, $x=-2$; $y=4-6+1$, $y=-2+1$, $y=-1$

D 3. $\frac{d}{dx}\left[\cos^2\left(x^3\right)\right]=$ $2(\cos x^3)(-\sin x^3)3x^2$ $-6x^2$

(A) $6x^2\sin\left(x^3\right)\cos\left(x^3\right)$ (B) $6x^2\cos\left(x^3\right)$ (C) $\sin^2\left(x^3\right)$ (D) $-6x^2\sin\left(x^3\right)\cos\left(x^3\right)$ (E) $-2\sin\left(x^3\right)\cos\left(x^3\right)$

B 4. An equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point $(1,5)$ is $\frac{(3x-2)(2)-(2x+3)(3)}{(3x-2)^2}$

(A) $13x-y=8$ (B) $13x+y=18$ (C) $x-13y=64$ (D) $x+13y=66$ (E) $-2x+3y=13$

$2-15=-13$; $y=5-13(x-1)$; $y=5-13x+13$; $13x+y=18$

A 5. If $f(x)=\ln\left(x+4+e^{-3x}\right)$, then $f'(0)$ is

$\frac{1-3e^{-3x}}{x+4+e^{-3x}}$ $\frac{-2}{5}$ (A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

E 6. If $y=x^2\sin(2x)$, then $\frac{dy}{dx}=$

$2x\sin 2x+x^2\cdot 2\cos 2x$; $2x(\sin 2x+x\cos 2x)$

(A) $2x\cos(2x)$ (B) $4x\cos(2x)$ (C) $2x\left[\sin(2x)+\cos(2x)\right]$

(D) $2x\left[\sin(2x)-x\cos(2x)\right]$ (E) $2x\left[\sin(2x)+x\cos(2x)\right]$

B 7. The $\lim\limits_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

B 8. What is the slope of the line tangent to the curve $3y^2-2x^2=6-2xy$ at the point $(3,2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$6yy'-4x=-2y-2xy'$; $12y'-12=-4-6y'$; $18y'=8$ $y'=\frac{4}{9}$

B 9. Let f be the function defined by $f(x)=x^3+x$. If $g(f(x))=x=f(g(x))$ and $g(2)=1$, what is the value of $g'(2)$? $f'=3x^2+1$, $f'(1)=4$

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

Part Los Dos: Frei Response.

10. (1992 AB4/BC1) Consider the curve defined by the equation $y+\cos y = x+1$, for $0 \le y \le 2\pi$.

(a) Find $\dfrac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve. Show the work that leads to your answer.

(c) Show that $\dfrac{d^2y}{dx^2} = \dfrac{\cos y}{(1-\sin y)^3}$, then find the values of y for which $\dfrac{d^2y}{dx^2} < 0$

(a) $\dfrac{dy}{dx} - \sin y \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx}\left[1 - \sin y\right] = 1$

$\dfrac{dy}{dx} = \dfrac{1}{1-\sin y}$

(b) Vert tangent when

$1 - \sin y = 0$

$\sin y = 1$

$\boxed{y = \frac{\pi}{2}}$

when $y = \frac{\pi}{2}$: $\frac{\pi}{2} + \cos\frac{\pi}{2} = x+1$

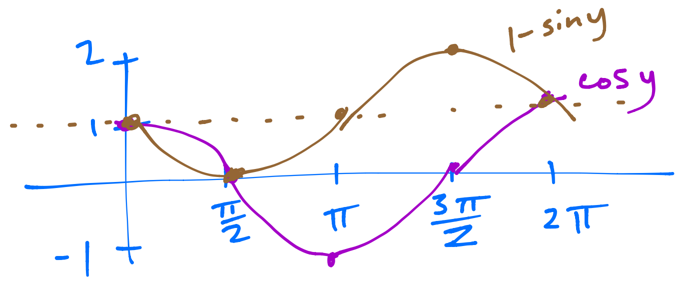
$\boxed{x = \frac{\pi}{2} - 1}$

(c) $\dfrac{dy}{dx} = (1-\sin y)^{-1}$

$\dfrac{d^2y}{dx^2} = -(1-\sin y)^{-2}\cdot(-\cos y)\cdot\dfrac{dy}{dx}$

$\dfrac{d^2y}{dx^2} = \dfrac{\cos y}{(1-\sin y)^2}\cdot\dfrac{1}{(1-\sin y)}$

$\boxed{\dfrac{d^2y}{dx^2} = \dfrac{\cos y}{(1-\sin y)^3}}$



$\dfrac{d^2y}{dx^2} < 0$ when $\cos y$ & $1-\sin y$ are opposite signs. This happens for $\frac{\pi}{2} < y < \frac{3\pi}{2}$

So $\boxed{\dfrac{d^2y}{dx^2} < 0 \text{ for } \frac{\pi}{2} < y < \frac{3\pi}{2}}$