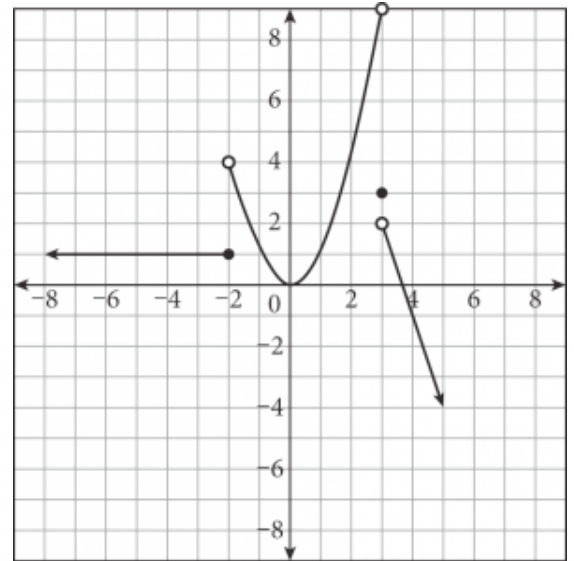Name_____ Date_____ Carbonated Drink_____

AP Calculus TEST: 1.1-1.5—Limits and Continuity. No Calculator

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.

Use the graph of the function $f(x)$ **shown at right to answer questions 1-2.**

_____ 1. What's the smallest value of k such that $f(x)$ is continuous on the interval $[k,3)$?

(A) −2   (B) −1   (C) −3   (D) −1.9   (E) No such value exists

_____ 2. What's the largest value of b such that $f(x)$ is continuous on $[-7,b]$ but not on $[-7,b+1]$?

(A) −3   (B) −2   (C) 4   (D) 1   (E) No such value exists

_____ 3. A function $f(x)$ is continuous for all x. The function satisfies the following:

$$f(1)=10, \ f(2)=3, \ f(3)=-5, \text{ and } f(4)=-18$$

The IVT says that the equation

(A) $f(x)=8.675309$ has a solution for some x with $x<-18$.

(B) $f(x)=8.675309$ has a solution for some $x \in (3,4)$.

(C) $f(x)=8.675309$ has a solution for some $x \in (2,3)$.

(D) $f(x)=8.675309$ has a solution for some $x \in (1,2)$.

(E) It cannot be determined from the information whether $f(x)=8.675309$ has a solution.

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_____ 4. $f(x)=\begin{cases} \dfrac{x^2+1}{x-1}, & x<0 \\ 2x-1, & 0 \le x \le 3 \\ \sqrt{x+1}, & x>3 \end{cases}$

Let $f(x)$ be defined by the piecewise equation above, then $f(x)$ is continuous

(A) for all real numbers   (B) for all $x \neq 0$ (C) for all $x \neq 3$ (D) for all $x \neq 0, 3$ (E) for all $x \neq 0,1,$ or 3

_____ 5. $\displaystyle \lim_{x \to 8} \frac{\frac{4}{x}-\frac{1}{2}}{x-8} =$ (A) DNE   (B) −16   (C) 16   (D) $\dfrac{1}{16}$ (E) $-\dfrac{1}{16}$

_____ 6. Evaluate $\lim\limits_{x \to 0} \left(\dfrac{3\csc 9x}{2\csc 3x} + \dfrac{x}{x} - \dfrac{\tan x}{\cos x + 1} \right) =$ (A) DNE   (B) 0   (C) $\dfrac{11}{2}$ (D) $\dfrac{3}{2}$ (E) 3

_____ 7. If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$, find the value of k that makes $f(x)$ continuous at $x = 2$.

(A) 0   (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{3}$ (D) 1   (E) $\dfrac{7}{5}$

_____ 8. $\lim\limits_{x \to 2} \dfrac{x^3 - x^2 - 3x + 2}{x^2 - 5x + 6} =$

(A) −5   (B) 5   (C) ∞   (D) $\dfrac{1}{3}$ (E) $-\dfrac{1}{3}$

_____ 9. The function $g(x) = \ln\left(x^2 - 1\right)$ is continuous for which values of x ?

(A) $-1 < x < 1$ (B) $-1 \leq x \leq 1$ (C) $x \leq -1$ or $x \geq 1$ (D) $x < -1$ or $x > 1$ (E) $x > 1$

Part II: Free Response: Show all work in the space provided. Be sure to use proper notation, notation, notation. No notation, No no points!!!

$$
\text{Let } f(x) = \begin{cases}
\dfrac{(2+x)^2 - 2(2+x) - 15}{x+5}, & x \le -3 \\[4mm]
\dfrac{\tan^2 2x}{3x^2}, & -3 < x \le \dfrac{1}{2} \\[4mm]
2x - a, & \dfrac{1}{2} < x < 1 \\[2mm]
3, & x = 1 \\[2mm]
bx^2 + a, & 1 < x < 2 \\[2mm]
\sqrt{x+2}, & 2 \le x \le 7 \\[2mm]
\dfrac{1}{2}x - \dfrac{1}{2}, & 7 < x \le 8 \\[4mm]
\dfrac{-5x^5 + 2x^2 + 7x + 14}{\sqrt{25x^{12} + 4x^4 + 13x^2 + 11}}, & x > 8
\end{cases}
$$

(a) Find $\displaystyle\lim_{x \to -5} f(x)$

(b) Find $\displaystyle\lim_{x \to \infty} f(x)$

(c) $\displaystyle\lim_{x \to 0} f(x) =$

(d) Find all values of a and b that make f continuous at $x=1$. Show all steps, and use correct notation, notation, notation.

(e) Does the IVT apply to $f(x)$ on $[7,8]$? Why or why not?