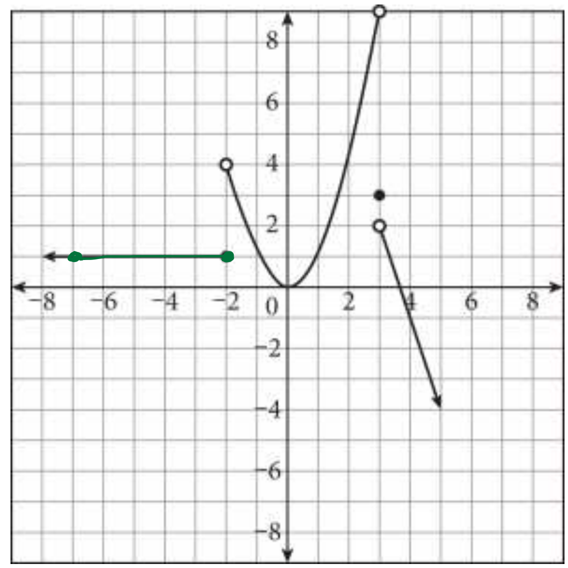
Name KEY Date________________ Carbonated Drink___________

AP Calculus TEST: 1.1-1.5—Limits and Continuity. No Calculator

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.

Use the graph of the function $f(x)$ shown at right to answer questions 1-2.



E 1. What's the smallest value of k such that $f(x)$ is continuous on the interval $[k,3)$? it would be the 1st real number to the right of $x=-2$, which doesn't exist.

(A) -2 (B) -1 (C) -3 (D) -1.9 (E) No such value exists

B 2. What's the largest value of b such that $f(x)$ is continuous on $[-7,b]$ but not on $[-7,b+1]$?

(A) -3 (B) -2 (C) 4 (D) 1 (E) No such value exists

D 3. A function $f(x)$ is continuous for all x. The function satisfies the following:

$$f(1)=10,\ f(2)=3,\ f(3)=-5, \text{ and } f(4)=-18$$

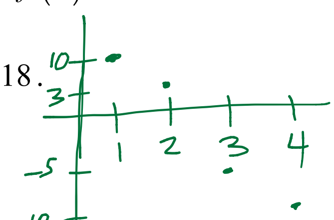
The IVT says that the equation

(A) $f(x)=8.675309$ has a solution for some x with $x<-18$.

(B) $f(x)=8.675309$ has a solution for some $x\in(3,4)$.

(C) $f(x)=8.675309$ has a solution for some $x\in(2,3)$.

(D) $f(x)=8.675309$ has a solution for some $x\in(1,2)$.



$f(2) < 8.675309 < f(1)$

(E) It cannot be determined from the information whether $f(x)=8.675309$ has a solution.

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C 4. $$f(x)=\begin{cases}\dfrac{x^2+1}{x-1}, & x<0\\ 2x-1, & 0\le x\le 3\\ \sqrt{x+1}, & x>3\end{cases}$$

Let $f(x)$ be defined by the piecewise equation above, then $f(x)$ is continuous

(A) for all real numbers (B) for all $x\neq 0$ (C) for all $x\neq 3$ (D) for all $x\neq 0,\ 3$ (E) for all $x\neq 0,1,$ or 3

E 5. $\lim\limits_{x\to 8}\dfrac{\frac{4}{x}-\frac{1}{2}}{x-8}\cdot\dfrac{2x}{2x}$ (A) DNE (B) -16 (C) 16 (D) $\dfrac{1}{16}$ (E) $-\dfrac{1}{16}$

$\lim\limits_{x\to 8}\dfrac{4(2)-(1)(x)}{2x(x-8)}$

$\lim\limits_{x\to 8}\dfrac{-\cancel{(x-8)}}{2x\cancel{(x-8)}}$

$\dfrac{-1}{16}$

D 6. Evaluate $\lim_{x\to 0}\left(\frac{3\csc 9x}{2\csc 3x}+\frac{x}{x}-\frac{\tan x}{\cos x+1}\right)=$ (A) DNE (B) 0 (C) $\frac{11}{2}$ (D) $\frac{3}{2}$ (E) 3

$\left(\frac{3}{2}\right)\left(\frac{3}{9}\right)+1-\frac{0}{1+1}$

$\frac{9}{18}+1-0$

$\frac{1}{2}+1$

$\frac{3}{2}$

B 7. If $f(x)=\begin{cases}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$, find the value of k that makes $f(x)$ continuous at $x=2$.

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

$\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\cdot\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$

$\lim_{x\to 2}\frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$

$\lim_{x\to 2}\frac{\cancel{(x-2)}\ 1}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$

$\frac{1}{6}$

A 8. $\lim_{x\to 2}\frac{x^3-x^2-3x+2}{x^2-5x+6}=$ $\frac{0}{0}$

(A) -5 (B) 5 (C) ∞ (D) $\frac{1}{3}$ (E) $-\frac{1}{3}$

$\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+x-1)}{\cancel{(x-2)}(x-3)}$

$\frac{2^2+2-1}{2-3}$

$\frac{5}{-1}$

-5

$$\begin{array}{r|rrrr} & 1 & -1 & -3 & 2\\ 2 & \downarrow & 2 & 2 & -2\\ \hline & 1 & 1 & -1 & \boxed{0}\end{array}$$

D 9. The function $g(x)=\ln\left(x^2-1\right)$ is continuous for which values of x?

(A) $-1<x<1$ (B) $-1\le x\le 1$ (C) $x\le -1$ or $x\ge 1$ (D) $x<-1$ or $x>1$ (E) $x>1$

$x^2-1>0$

$(x-1)(x+1)>0$

pos pos

-1 1

$x<-1$ or $x>1$

Part II: Free Response: Show all work in the space provided. Be sure to use proper notation, notation, notation. No notation, No no points!!!

$$\text{Let } f(x) = \begin{cases} \dfrac{(2+x)^2 - 2(2+x) - 15}{x+5}, & x \le -3 \\ \dfrac{\tan^2 2x}{3x^2}, & -3 < x \le \dfrac{1}{2} \\ 2x - a, & \dfrac{1}{2} < x < 1 \\ 3, & x = 1 \\ bx^2 + a, & 1 < x < 2 \\ \sqrt{x+2}, & 2 \le x \le 7 \\ \dfrac{1}{2}x - \dfrac{1}{2}, & 7 < x \le 8 \\ \dfrac{-5x^5 + 2x^2 + 7x + 14}{\sqrt{25x^{12} + 4x^4 + 13x^2 + 11}}, & x > 8 \end{cases}$$

(a) Find $\lim_{x \to -5} f(x)$

Process

$$\lim_{x\to -5} \frac{(2+x)^2 - 2(2+x) - 15}{x+5}$$

$$\lim_{x\to -5} \frac{\cancel{4} + 4x + x^2 - \cancel{4} - 2x - 15}{x+5}$$

$$\lim_{x\to -5} \frac{x^2 + 2x - 15}{x+5}$$

√1

$$\lim_{x\to -5} \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}}$$

$-5-3$

-8

√2

(b) Find $\lim_{x \to \infty} f(x)$

$$\lim_{x\to\infty} \frac{-5x^5 + 2x^2 + 7x + 14}{\sqrt{25x^{12} + 4x^4 + 13x^2 + 11}}$$

$$\approx \lim_{x\to\infty} \frac{-5x^5 + \cdots}{\sqrt{25x^{12} + \cdots}}$$

$$\lim_{x\to\infty} \frac{-5x^5 + \cdots}{5x^6 + \cdots}$$

0

√3

(c) $\lim_{x \to 0} f(x) =$

$$\lim_{x\to 0} \frac{\tan^2 2x}{3x^2}$$

$$\lim_{x\to 0} \left(\frac{1}{3}\right)\left(\frac{\tan 2x}{1x}\right)\left(\frac{\tan 2x}{1x}\right)$$

$$\left(\frac{1}{3}\right)\left(\frac{2}{1}\right)\left(\frac{2}{1}\right)$$

$\frac{4}{3}$

√4

(d) Find all values of a and b that make f continuous at $x = 1$. Show all steps, and use correct notation, notation, notation.

$$f(x) = \begin{cases} 2x - a, & \frac{1}{2} < x < 1 \\ 3, & x = 1 \\ bx^2 + a, & 1 < x < 2 \end{cases}$$

$\lim_{x \to 1^-} f(x) = 2 - a$

$f(1) = 3$

$\lim_{x \to 1^+} f(x) = b + a$

process

5 √

So, $2 - a = 3$ & $b + a = 3$

$-a = 1$

$\boxed{a = -1}$ So, $b - 1 = 3$

$\boxed{b = 4}$

6 √ √7

(e) Does the IVT apply to $f(x)$ on $[7,8]$? Why or why not?

$$f(x) = \begin{cases} \sqrt{x+2}, & 2 \le x \le 7 \\ \frac{1}{2}x - \frac{1}{2}, & 7 < x \le 8 \end{cases}$$

$\lim_{x \to 7^+} f(x) = \frac{1}{2}(7) - \frac{1}{2}$

$= \frac{7}{2} - \frac{1}{2}$

$= \frac{6}{2}$

$= 3$

$f(7) = \sqrt{7+2}$

$= \sqrt{9}$

$= 3$

So, $f(x)$ is continuous at $x = 7$ (left endpoint)

Since the line $y = \frac{1}{2}x - \frac{1}{2}$ is continuous on $x \in (7, 8]$,

* The IVT does apply to $f(x)$ on $[7, 8]$, since $f(x)$ is continuous on $[7, 8]$. √8 √9