

Name

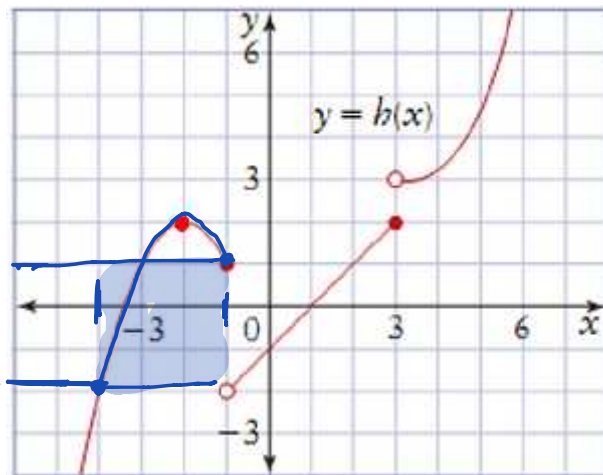
KEY

Date

Period

AP Calculus TEST: 1.1-1.5

No Calculator

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.**Use the graph of the function $h(x)$, shown below right, to answer questions 1-3.**

- D 1. The smallest value of $a \in \mathbb{R}$ such that $h(x)$ is continuous on $[a, 3]$ is *No 1st real number to right of $x = -1$*

(A) 0 (B) -1 (C) -0.9 (D) No such value exists

- D 2. On the interval $-4 \leq x \leq -1$, the IVT guarantees a value $-4 < k < -1$ such that $h(k) = P$. According to the IVT, which of the following of P is NOT guaranteed?

(A) 0 (B) $\frac{1}{2}$ (C) -1 (D) 2

- A 3. $\lim_{x \rightarrow -1^-} h(h(x)) = h(\lim_{x \rightarrow -1^-} h(x)) = h(-2) = 0$

(A) 0 (B) 1 (C) 2 (D) -2

- B 4. The line $y = -5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = e^{-x} + 5$ (B) $y = \frac{25x^3 + 2x - 1}{\sqrt{25x^6 + 50}}$ (C) $y = \frac{50x^3 - 2x^2 - 7}{7 + 9x + 10x^3}$ (D) $y = -\frac{\sin(10x)}{5x}$

$$y \approx \frac{25x^3}{5x^3} \rightarrow \text{HA @ } y = 5$$

- A 5. $\lim_{x \rightarrow -1} \frac{2 - \sqrt{x+5}}{(x-1)(x+1)} = \frac{2 + \sqrt{x+5}}{2 + \sqrt{x+5}}$ (A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2}$

$$\lim_{x \rightarrow -1} \frac{(4 - (x+5))}{(x-1)(x+1)(2 + \sqrt{x+5})}$$

$$\lim_{x \rightarrow -1} \frac{-x-1}{(x-1)(x+1)(2 + \sqrt{x+5})}$$

$$\lim_{x \rightarrow -1} \frac{(-1)(x+1)}{(x-1)(x+1)(2 + \sqrt{x+5})}$$

$$\frac{-1}{(-2)(2+2)}$$

$$= \frac{-1}{-8}$$

$$= \frac{1}{8}$$

6. $\lim_{x \rightarrow 5} \frac{\frac{1}{x+2} - \frac{1}{7}}{x-5}$ (A) -1 (B) $\frac{1}{49}$ (C) $-\frac{1}{49}$ (D) $\frac{1}{7}$

$$\lim_{x \rightarrow 5} \frac{7 - (x+2)}{(x-5)(7)(x+2)}$$

$$\lim_{x \rightarrow 5} \frac{-x+5}{(x-5)(7)(x+2)}$$

$$\lim_{x \rightarrow 5} \frac{(-1)(x-5)}{(x-5)(7)(x+2)}$$

$$\frac{-1}{7(7)} = -\frac{1}{49}$$

7. Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{(x-1)^3}{x^3-1} + \frac{4 \tan 3x}{3 \tan 4x} - \frac{x}{|x|} \right) =$ (A) 1 (B) 3 (C) 8 (D) $\frac{16}{9}$

$$\frac{(-1)^3}{-1} + \frac{4}{3} \left(\frac{3}{4} \right) - \frac{2}{12}$$

$$= -1 + 1 - 1$$

$$= -1$$

$$f(x) = \begin{cases} \frac{x^2 + \sin^2 2x}{x^2}, & x \neq 0 \\ b, & x = 0 \end{cases}$$

8. Let f be the function defined above. For what value of b is f continuous at $x = 0$? (A) 2 (B) 3 (C) 5 (D) no such value exists

$$\lim_{x \rightarrow 0} \frac{x^2 + \sin^2 2x}{x^2}$$

$$\lim_{x \rightarrow 0} \left[\frac{x^2}{x^2} + \frac{(\sin 2x)^2}{x^2} \right]$$

$$\lim_{x \rightarrow 0} \left[1 + \frac{\sin 2x}{2x} \cdot \frac{\sin 2x}{2x} \left(\frac{2}{1} \right) \right]$$

$$1 + 1 \cdot 1 \cdot 4 = 5$$

9. The function f is continuous on $[-4, 5]$ and has values given in the table below. The equation $f(x) = 6$ at least two solutions in the interval $(-4, 5)$ if $p =$

x	-4	0	5
$f(x)$	1	p	4

(A) 6 (B) 6.1 (C) 5.9 (D) 5

$$p > 6$$

Part II: Free Response: Answer all questions in the space provided.. Show all steps on part (e), and all parts, use proper notation, notation, notation. No Notation, No-No point!!

10. Let $f(x)$ be the totally awesome piece wise function given below.

$$f(x) = \begin{cases} \frac{2x^5 + 7x^3 - 2x + 1}{\sqrt{9x^{12} + 2x^4 + 11}}, & x \leq -3 \\ ax^2 - b, & -3 < x < -1 \\ 10, & x = -1 \\ 2ax - 3b, & -1 < x < -\frac{1}{2} \\ \frac{x^2 - 4x + 3}{x^2 |2x - 2|}, & -\frac{1}{2} \leq x < 3 \\ \frac{(x-3)^2}{4-x}, & x \geq 3 \end{cases}$$

(a) Find $\lim_{x \rightarrow -\infty} f(x) = 0$ (✓1)

$\approx \lim_{x \rightarrow -\infty} \frac{2x^5}{3x^6} = 0$

(b) Find $\lim_{x \rightarrow 1^-} f(x) = 1$ (✓2)

$\lim_{x \rightarrow 1^-} \frac{(x-3)(x-1)}{x^2 |2x-2|}$

$\left[\frac{-2}{1} \right] \left[\frac{-1}{1-2} \right]$

$-2(-\frac{1}{2}) = 1$

(c) Find $\lim_{x \rightarrow 4^+} f(x) = \text{DNE or } -\infty$ (✓3)

(d) Is $f(x)$ continuous at $x = 3$? Justify using the 3-step definition of continuity at a point.

$$\lim_{x \rightarrow 3^-} f(x) = \frac{9 - 12 + 3}{9|4|} = \frac{0}{18} = 0$$

$$f(3) = \frac{0}{1} = 0$$

$$\lim_{x \rightarrow 3^+} f(x) = 0$$

$f(x)$ is continuous at $x = 3$

Since $0 = 0 = 0$.

✓4

✓5

✓6

(e) If a and b are constants that make $f(x)$ continuous at $x = -1$, what is the value of a ?

$$\lim_{x \rightarrow -1^-} f(x) = a - b$$

$$f(-1) = 10$$

$$\lim_{x \rightarrow -1^+} f(x) = -2a - 3b$$

$$\text{So } a - b = 10 \rightarrow a = 10 + b$$

$$-2a - 3b = 10$$

$$-2(10 + b) - 3b = 10$$

$$-20 - 2b - 3b = 10$$

$$-5b = 30$$

$$b = -6$$

$$\text{So, } a = 10 - 6$$

$$a = 4$$

✓7

✓8

✓9