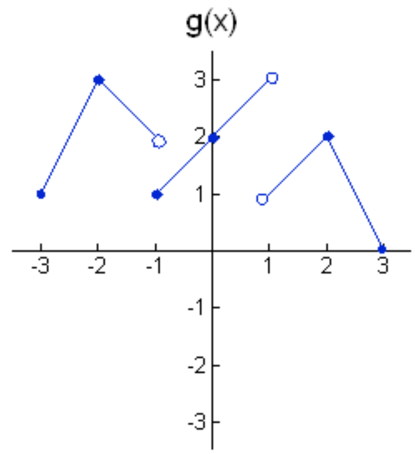
Name________________________________ Date____________________ Period__________

AP Calculus TEST: 3.1-3.5 No Calculator



Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.

Use the graph of the function $g(x)$ shown at right to answer questions 1-2.

__E__ 1. The smallest value of $a \in \mathbb{R}$ such that $g(x)$ is continuous on $[a,3]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) No such value exists

__E__ 2. Find the number $x=b$ such that $g(x)$ is continuous in $(-1,b)$ but not in $[-1,b]$.

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 0.999999 (E) 1

__D__ 3. If $g(x)=\cos x$, then on the interval $\left[\frac{7\pi}{6},\frac{7\pi}{4}\right]$, by the IVT, $g(x)$ MUST equal what value for some $x \in \left(\frac{7\pi}{6},\frac{7\pi}{4}\right)$?

(A) -1 (B) 1 (C) $\frac{4\pi}{3}$ (D) 0 (E) $\frac{\sqrt{3}}{2}$

__A__ 4. $\lim\limits_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2}=$ $\frac{x^2(5x^2+8)}{x^2(3x^2-16)}$ (A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

__E__ 5. $\lim\limits_{x\to 8}\frac{\frac{4}{x}-\frac{1}{2}}{x-8}$ $\left(\frac{2x}{2x}\right)$ $=\frac{(8-x)\cdot 1}{(x-8)2x}$ (A) DNE (B) -16 (C) 16 (D) $\frac{1}{16}$ (E) $-\frac{1}{16}$

__D__ 6. Evaluate $\lim\limits_{x\to 0}\left(\frac{3\sin 3x}{2\sin 9x}+\frac{x}{x}-\frac{\tan x}{\cos x+1}\right)=$ (A) DNE (B) 0 (C) $\frac{11}{2}$ (D) $\frac{3}{2}$ (E) 3

$\frac{1}{2}\cdot 3+1-0=\frac{3}{2}$

__B__ 7. If $f(x)=\begin{cases}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$ $\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$ $=\frac{(2x+5-x-7)\,1}{(x-2)\;3+3}$ find the value of k that makes $f(x)$ continuous at $x=2$.

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

__E__ 8. The function f is continuous on $[-3,2]$ and has values given in the table below. If the equation $f(x)=2$ has at least 2 solutions in the interval $(-3,2)$ if $k=$

x	-3	0	2
$f(x)$	5	k	3.2

(A) 5 (B) 3.2 (C) 2 (D) 10 (E) -3

16 TOTAL CHECKS

1 E
2 E
3 D
4 A
5 E
6 D
7 B
8 E

Part II: Free Response: Answer all questions below the given line. **Show all steps, label parts, and write legibly.**

9. Let $f(x)=\begin{cases} \dfrac{(1+x)^2+2(1+x)-3}{x+4}, & x\le -3 \\ 2x-a, & -3<x<1 \\ 3, & x=1 \\ bx^2+a, & 1<x<555 \\ \dfrac{-4x^3+2x^2+7x+14}{\sqrt{16x^6+4x^4+13x^2+11}}, & x\ge 555 \end{cases}$

(a) Find $\lim\limits_{x\to -4} f(x)$

(b) Find $\lim\limits_{x\to\infty} f(x)$

(c) Find all values of a and b that make f continuous at $x=1$. Show all steps, and use correct notation, notation, notation.

(a) $\lim\limits_{x\to -4} f(x) = \lim\limits_{x\to -4} \dfrac{(1+x)^2+2(1+x)-3}{x+4}$

$= \lim\limits_{x\to -4} \dfrac{\cancel{1}+2x+x^2+\cancel{2}+2x\cancel{-3}}{x+4}$

$= \lim\limits_{x\to -4} \dfrac{x^2+4x}{x+4} = \lim\limits_{x\to -4} \dfrac{x\cancel{(x+4)}}{\cancel{(x+4)}}$

$= \boxed{-4}$

(b) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{-4x^3+\cdots}{\sqrt{16x^6+\cdots}}$

$= \boxed{-1}$

(c) $\boxed{\lim\limits_{x\to 1^-} f(x) = 2-a}$

$\boxed{\lim\limits_{x\to 1^+} f(x) = b+a}$

$\boxed{f(1) = 3}$

so $\boxed{\begin{cases} -a+2=3 \\ a+b=3 \end{cases}}$

$\boxed{a=-1}$

so $-1+b=3$

$\boxed{b=4}$

8 checks

2. Evaluate the following. For each, show all steps and work. Careful rewriting the "lim" each time!!! Part e) doesn't require any work.

a) $\lim_{x\to0}\frac{\tan 2x + x}{5x} =$ $\frac{0}{0}$

$\lim_{x\to0}\left[\frac{(\tan 2x)\cdot 2}{5x\cdot 2}+\frac{\cancel{x}}{5\cancel{x}}\right]$

$\lim_{x\to0}\left[\left(\frac{2}{5}\right)\left(\frac{\tan 2x}{2x}\right)+\frac{1}{5}\right]$

$\left(\frac{2}{5}\right)(1)+\frac{1}{5}$

$\boxed{\frac{3}{5}}$

b) $\lim_{x\to0}\frac{4x\sin x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}$

$\lim_{x\to0}\frac{4x\sin x(1+\cos x)}{1-\cos^2 x}$

$\lim_{x\to0}\frac{4x\sin x\cdot(1+\cos x)}{\sin^2 x}$

$\lim_{x\to0}\left[\frac{x}{\sin x}\cdot\frac{4\sin x(1+\cos x)}{\sin x}\right]$

$1\cdot4\cdot(1+1)$

$\boxed{8}$

c) $\lim_{x\to-2}\frac{x^2-4}{\sqrt{6+x}-2}\cdot\left(\frac{\sqrt{6+x}+2}{\sqrt{6+x}+2}\right)$ $\frac{0}{0}$

$\lim_{x\to-2}\frac{(x-2)(x+2)(\sqrt{6+x}+2)}{6+x-4}$

$\lim_{x\to-2}\frac{(x-2)\cancel{(x+2)}(\sqrt{6+x}+2)}{\cancel{(x+2)}}$

$(-4)(2+2)$

$\boxed{-16}$

d) $\lim_{x\to3}\frac{\frac{2}{x+2}-\frac{2}{5}}{x-3}\left(\frac{5(x+2)}{5(x+2)}\right)$ $\frac{0}{0}$

$\lim_{x\to3}\frac{2(5)-2(x+2)}{5(x-3)(x+2)}$

$\lim_{x\to3}\frac{10-2x-4}{5(x-3)(x+2)}$

$\lim_{x\to3}\frac{6-2x}{5(x-3)(x+2)}$

$\lim_{x\to3}\frac{-2\cancel{(x-3)}}{5\cancel{(x-3)}(x+2)}$

$\boxed{\frac{-2}{25}}$

e) $\lim_{x\to-\infty}\frac{4x^5+2x^2-3x+1}{\sqrt{9x^{10}+11x^9+12x^2+13x+14}}=$

$\approx\lim_{x\to-\infty}\frac{4x^5+\cdots}{3x^5+\cdots}$

$=-\frac{4}{3}$

plug in negative into original leading terms to get sign

f) $\lim_{x\to5^+}\frac{x^2|10-2x|}{\sin\left(\frac{x\pi}{6}\right)(3x^2-18x+15)}=$ $\frac{0}{0}$

$\lim_{x\to5^+}\frac{x^2|10-2x|}{\sin\left(\frac{x\pi}{6}\right)(3)(x^2-6x+5)}$

$\lim_{x\to5^+}\frac{x^2|10-2x|}{\sin\left(\frac{x\pi}{6}\right)(3)(x-5)(x-1)}$

$\lim_{x\to5^+}\frac{x^2}{3\sin\left(\frac{x\pi}{6}\right)(x-1)}\cdot\frac{|10-2x|}{x-5}$ Jump! Man!

$\frac{25}{3\cdot\sin\frac{5\pi}{6}\cdot4}\cdot\left(\frac{2}{1}\right)$ plug in a number bigger than 5

$\frac{25}{12\cdot\left(\frac{1}{2}\right)}(2)$

$\frac{25\cdot2}{6}$

$\boxed{\frac{25}{3}}$