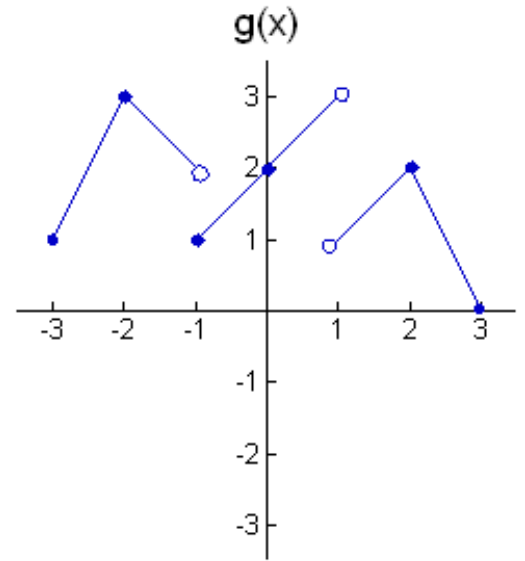


AP Calculus TEST: Limits and Continuity
No Calculator

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.

Use the graph of the function $g(x)$ shown at right to answer questions 1-3.



_____ 1. $\lim_{x \rightarrow -1^+} g(x^2) + \lim_{x \rightarrow -2} [g(x)]^2 + g(-1) =$
 (A) 10 (B) 11 (C) 12 (D) 13 (E) DNE

_____ 2. $\lim_{x \rightarrow 3^-} g(g(x)) =$
 (A) 0 (B) 3 (C) 2 (D) 1 (E) DNE

_____ 3. Find the number $x = b$ such that $g(x)$ is continuous in $(-1, b)$ but not in $[-1, b]$.
 (A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 0.999999 (E) 1

_____ 4. Evaluate $\lim_{x \rightarrow 0} \frac{3 \cot 6x}{2 \csc 2x} + 1$
 (A) DNE (B) 0 (C) $\frac{11}{2}$ (D) $\frac{3}{2}$ (E) 3

_____ 5. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x + 1}$
 (A) DNE (B) 0 (C) 1 (D) -1 (E) 4

_____ 6. If $f(x) = \begin{cases} 2x^2 + 1, & x < -1 \\ -\frac{3}{x}, & x \geq -1 \end{cases}$, which of the following is NOT true?
 (A) $\lim_{x \rightarrow -1^+} f(x) = f(-1)$ (B) $f(x)$ is continuous at $x = -1$
 (C) $\lim_{x \rightarrow \infty} f(x) = 0$ (D) $f(x)$ has a vertical asymptote at $x = 0$ (E) $\lim_{x \rightarrow -1^-} f(x) = -1$

_____ 7. If $\sec x \leq M(x) \leq e^x$, for all x in an interval containing $x = 0$, then $\lim_{x \rightarrow 0} M(x) =$
 (A) DNE (B) 0 (C) 1 (D) -1 (E) Not enough information

_____ 9. If $g(x) = \cos x$, then on the interval $\left[\frac{7\pi}{6}, \frac{7\pi}{4}\right]$, by the IVT, $g(x)$ MUST equal what value for some $x \in \left(\frac{7\pi}{6}, \frac{7\pi}{4}\right)$?
 (A) -1 (B) 1 (C) $\frac{4\pi}{3}$ (D) 0 (E) $\frac{1}{2}$

Part II: Free Response: Answer all questions below the given line. **Show all steps, label parts, and write legibly.**

1. Let $f(x) = \begin{cases} 2x - a, & x < 1 \\ 3, & x = 1 \\ bx^2 + a, & x > 1 \end{cases}$

- a. Find $f(3)$
 - b. Find $\lim_{x \rightarrow 1^-} f(x)$
 - c. Find $\lim_{x \rightarrow 1^+} f(x)$
 - d. Find all values of a and b that make f continuous at $x = 1$. Justify your answer.
 - e. Find $\lim_{x \rightarrow -\infty} f(x)$
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2. Evaluate the following. For each, show all steps and work. Careful rewriting the “lim” each time!!! Part e) doesn't require any work.

$$\text{a) } \lim_{x \rightarrow 0} \frac{\tan 2x + x}{5x} =$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{4x \sin x}{1 - \cos x} =$$

$$\text{c) } \lim_{x \rightarrow -2} \frac{x^2 - 4}{\sqrt{6 + x} - 2} =$$

$$\text{d) } \lim_{x \rightarrow 3} \frac{\frac{2}{x+2} - \frac{2}{5}}{x-3} =$$

$$\text{e) } \lim_{x \rightarrow -\infty} \frac{4x^5 + 2x^2 - 3x + 1}{\sqrt{9x^{10} + 11x^9 + 12x^2 + 13x + 14}} =$$

$$\text{e) } \lim_{x \rightarrow 5^+} \frac{x^2 |10 - 2x|}{\sin\left(\frac{x\pi}{6}\right) (3x^2 - 18x + 15)} =$$