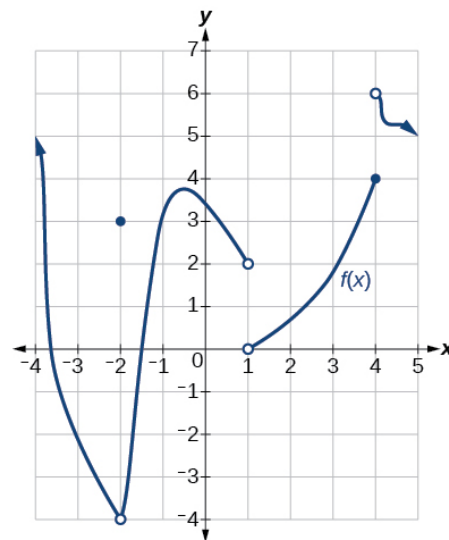


Name KEY Date Friday, 9/14/2018 Favorite Allergy Math

AP Calculus TEST 1.1-1.4, No Calculator, AB

Part I—Multiple Choice: Put the correct CAPITAL LETTER in the space provided next to each question number.



- E 1. Using the graph of $f(x)$ on the right (for this problem only), what is the value of
- $$\lim_{x \rightarrow -1^-} f(x^2) + \lim_{x \rightarrow 4^-} \sqrt{f(x)} - \lim_{x \rightarrow -2} f(x)$$
- (A) -2 (B) 0 (C) 2 (D) 7 (E) 8

$$\lim_{x \rightarrow -1^-} f(x^2) = \frac{f(x)}{2} + \sqrt{4} = \frac{-4}{2} + 2 = -2 + 2 = 0$$

- A 2. $\lim_{x \rightarrow 3^+} \frac{4-x}{(x-3)^2} = \frac{1}{0} + \infty$
- (A) ∞ (B) $-\infty$ (C) $\frac{4}{3}$ (D) $\frac{4}{9}$ (E) $-\frac{1}{9}$

plug in 3.1 for sign $\frac{+}{+} = +$

- B 3. $\lim_{x \rightarrow 7} \frac{3-\sqrt{x+2}}{7-x} = \frac{0}{0}$ (RATCON)
- (A) 1 (B) $\frac{1}{6}$ (C) $-\frac{1}{6}$ (D) -1 (E) DNE

$$\lim_{x \rightarrow 7} \frac{9-(x+2)}{(7-x)(3+\sqrt{x+2})}$$

$$\lim_{x \rightarrow 7} \frac{(7-x)1}{(7-x)(3+\sqrt{x+2})} = \frac{1}{6}$$

- A 4. $\lim_{x \rightarrow 4} \frac{2}{x+1} - \frac{2}{x-4} = \frac{0}{0}$ (LCM)
- (A) $-\frac{2}{25}$ (B) $\frac{2}{25}$ (C) DNE (D) $-\frac{25}{2}$ (E) $\frac{25}{2}$

$$\lim_{x \rightarrow 4} \frac{2(5) - 2(x+1)}{5(x-4)(x+1)} = \lim_{x \rightarrow 4} \frac{-2(x-4)}{5(x-4)(x+1)}$$

$$\lim_{x \rightarrow 4} \frac{10 - 2x - 2}{5(x-4)(x+1)} = \frac{-2}{25}$$

$$\lim_{x \rightarrow 4} \frac{-2x + 8}{5(x-4)(x+1)}$$

- E 5. If $e^x + \cos x \leq B(x) \leq \frac{5x^2 - 2x - 1}{\sec x}$, what is $\lim_{x \rightarrow 0} B(x)$? SQUEEZE THM

- (A) 2 (B) 1 (C) 5 (D) 3 (E) Not enough information

$$\lim_{x \rightarrow 0} (e^x + \cos x) = e^0 + \cos 0 = 1 + 1 = 2$$

$$\lim_{x \rightarrow 0} \frac{5x^2 - 2x - 1}{\sec x} = \frac{-1}{-1} = 1$$

$$\uparrow 2 \neq -1 \uparrow$$

D 6. $\lim_{x \rightarrow -3} \frac{x^3 - 3x^2 - 10x + 24}{x^2 + 4x + 3} = \frac{0}{0}$

- (A) ∞ (B) $\frac{-19}{2}$ (C) $\frac{1}{2}$ (D) $\frac{-35}{2}$ (E) $\frac{-17}{2}$

synthetic division on numerator

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

so, $\lim_{x \rightarrow -3} \frac{(x+3)(x^2 - 6x + 8)}{(x+3)(x+1)} = \frac{9 + 18 + 8}{-2} = \frac{35}{-2}$

E 7. $\lim_{x \rightarrow -\infty} \frac{-4x^5 - 4x^3 - 5x^2 - 1}{\sqrt{4x^{12} + 5x^4 + 19}} = \frac{-4x^5}{2x^6}$ Bigger on top $\rightarrow 0$

- (A) -1 (B) 1 (C) -2 (D) 2 (E) 0

C 8. $\lim_{x \rightarrow 0} \frac{(1+x)^2 - 3(1+x) + 2}{x} = \frac{0}{0}$

- (A) 0 (B) 1 (C) -1 (D) DNE (E) 2

$\lim_{x \rightarrow 0} \frac{1 + 2x + x^2 - 3 - 3x + 2}{x} = \frac{x^2 - x}{x}$

$\lim_{x \rightarrow 0} \frac{x^2 - x}{x} = \frac{x(x-1)}{x}$

$\lim_{x \rightarrow 0} \frac{x(x-1)}{x} = -1$

C 9. If $f(x) = \begin{cases} ax+b, & x < -2 \\ -3, & x = -2 \\ bx^2+a, & x > -2 \end{cases}$ is continuous at $x = -2$, what is the value of $a+b$?

- (A) 1 (B) -1 (C) 0 (D) -2 (E) 2

$\lim_{x \rightarrow -2^-} f(x) = -2a + b$

$f(-2) = -3$

$\lim_{x \rightarrow -2^+} f(x) = 4b + a$

$\begin{cases} -2a + b = -3 \\ a + 4b = -3 \end{cases}$

sub: $a = -4b - 3$

so, $-2(-4b - 3) + b = -3$

$8b + 6 + b = -3$

$9b = -9$

$b = -1$

so, $a = -4(-1) - 3$

$a = 4 - 3$

$a = 1$

so, $a+b = 1-1 = 0$

Part II—Free Response: Show all work in the space provided. Use proper notation.

Let a piecewise function be defined below.

$$f(x) = \begin{cases} \frac{1+7e^{-x}}{5-4e^{-x}}, & x < -8 \\ \frac{x+5}{x^2-25}, & -8 \leq x \leq -2 \\ 2x^2+4x-5, & -2 < x \leq 0 \\ \frac{x+5}{e^x + \ln(x+1)}, & 0 < x < 1 \\ \csc x, & 1 \leq x < 2\pi \\ (\arctan x)^3, & x \geq 2\pi \end{cases}$$

(a) Using the 3-step definition of continuity at a point, determine if $f(x)$ is continuous at $x = 0$.

either one $\downarrow$

(1) $\lim_{x \rightarrow 0^-} f(x) = -5$

$f(0) = -5$

(2) $\lim_{x \rightarrow 0^+} f(x) = \frac{5}{1} = 5$

f is not continuous at $x=0$ (3)

Since $-5 \neq 5$ (4)

(b) $\lim_{x \rightarrow -5^-} f(x) =$

$\lim_{x \rightarrow -5^-} f(x)$

$\lim_{x \rightarrow -5^-} \frac{x+5}{x^2-25}$

$\lim_{x \rightarrow -5^-} \frac{(x+5)1}{(x+5)(x-5)}$

$\frac{1}{-5-5}$ (5)

$\frac{1}{-10}$

(c) $\lim_{x \rightarrow -\infty} f(x) =$

$$\lim_{x \rightarrow -\infty} \frac{1+7e^{-x}}{5-4e^{-x}} = -\frac{7}{4} \quad (\sqrt{6})$$

$$\frac{1+7e^{\infty}}{5-4e^{\infty}} \approx \frac{7e^{\infty}}{-4e^{\infty}} \rightarrow -\frac{7}{4} \quad \left(\begin{array}{l} \text{grow at same} \\ \text{rate: } \frac{\text{leading coeff}}{\text{leading coeff}} \end{array} \right)$$

(d) $\lim_{x \rightarrow \infty} f(x) =$

$$\lim_{x \rightarrow \infty} (\arctan x)^3$$

$$\left(\lim_{x \rightarrow \infty} \arctan x \right)^3$$

$$\left(\frac{\pi}{2} \right)^3 \text{ or } \frac{\pi^3}{8} \quad (\sqrt{7})$$

(e) $\lim_{x \rightarrow \frac{5\pi}{4}} f(x) =$

$$\lim_{x \rightarrow \frac{5\pi}{4}} \csc x$$

$$\csc \frac{5\pi}{4} \text{ or } -\frac{2}{\sqrt{2}} \text{ or } -\sqrt{2} \quad (\sqrt{8})$$

Notation on all F.R. $(\sqrt{9})$

9 checks