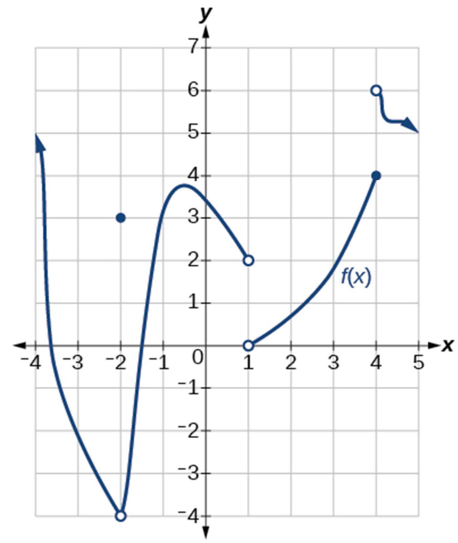
Name KEY Date Coin Denomination

AP Calculus TEST 1.1-1.4, No Calculator

Part I—Multiple Choice: Put the correct CAPTIAL LETTER in the space provided next to each question number.



D 1. Using the graph of $f(x)$ on the right (for this problem only), what is the value of

$$\lim_{x\to 4^-} f(x-3) + \lim_{x\to -2}\left[f(x)\right]^2 - \lim_{x\to 4^+} f(x)$$

(A) 5 (B) 7 (C) 10 (D) 12 (E) 26

$2 + 16 - 6 = 12$

A 2. $\lim_{x\to 2^-} \dfrac{x-5}{x-2} = \dfrac{-3}{0} \to +\infty$

(A) ∞ (B) $-\infty$ (C) $\dfrac{5}{2}$ (D) 1 (E) 0

B 3. $\lim_{x\to -1} \dfrac{\sqrt{x+5}-2}{x+1}$ $\frac{0}{0}$ $\cdot \dfrac{\sqrt{x+5}+2}{\sqrt{x+5}+2} = \lim_{x\to -1} \dfrac{(x+5-4)\,1}{(x+1)(\sqrt{x+5}+2)} = \dfrac{1}{4}$

(A) 1 (B) $\dfrac{1}{4}$ (C) $-\dfrac{1}{4}$ (D) -1 (E) DNE

D 4. $\lim_{x\to 5} \dfrac{x-5}{\dfrac{3}{x-1}-\dfrac{3}{4}}$ $\frac{0}{0}$ $\cdot \dfrac{4(x-1)}{4(x-1)} = \lim_{x\to 5} \dfrac{4(x-1)(x-5)}{12-3(x-1)} = \lim_{x\to 5} \dfrac{4(x-1)(x-5)}{-3x+3+12} = \lim_{x\to 5} \dfrac{4(x-1)\cancel{(x-5)}}{-3\cancel{(x-5)}} = \dfrac{4(4)}{-3} = \dfrac{-16}{3}$

(A) $-\dfrac{4}{3}$ (B) $\dfrac{4}{3}$ (C) DNE (D) $-\dfrac{16}{3}$ (E) $\dfrac{16}{3}$

D 5. $\lim_{x\to 0} \dfrac{\sin^2 2x}{\tan^2 5x}$ $\frac{0}{0}$ $= \lim_{x\to 0} \left[\dfrac{\sin 2x}{\tan 5x}\right]\left[\dfrac{\sin 2x}{\tan 5x}\right] = \left(\dfrac{2}{5}\right)\left(\dfrac{2}{5}\right) = \dfrac{4}{25}$

(A) ∞ (B) $\dfrac{2}{5}$ (C) $-\infty$ (D) $\dfrac{4}{25}$ (E) 0

D 6. $\lim_{x \to -\infty} \dfrac{8x^3+2x^2-14}{\sqrt{16x^6+11x^4+9}} = \dfrac{-8}{4} = -2$

(A) $-\dfrac{1}{2}$ (B) $\dfrac{1}{2}$ (C) 0 (D) -2 (E) 2

E 7. $\lim_{m \to 0} \dfrac{2(x+m)-x^2+(x+m)^2-2x}{m} = \frac{0}{0}$

(A) $2+x$ (B) 2 (C) $1+2x$ (D) DNE (E) $2+2x$

$\lim_{m\to 0} \dfrac{\cancel{2x}+2m\cancel{-x^2}\cancel{+x^2}+2xm+m^2\cancel{-2x}}{m}$

$\lim_{m\to 0} \dfrac{\cancel{m}(2+2x+m)}{\cancel{m}}$

$2+2x$

A 8. If $f(x) = \begin{cases} ax+b, & x<-1 \\ -3, & x=-1 \\ 2ax^2+bx, & x>-1 \end{cases}$ is continuous at $x=-1$, what is the value of $a \cdot b$?

(A) 54 (B) -15 (C) 3 (D) -9 (E) 28

$\lim_{x\to -1^-} f(x) = -a+b$

$f(-1) = -3$

$\lim_{x\to -1^+} f(x) = 2a-b$

so $\begin{array}{r} -a+b=-3 \\ +\ 2a-b=-3 \\ \hline a=-6 \end{array}$

so, $6+b=-3$

$b=-9$

so, $ab=(-6)(-9)=54$

C 9. If $2^x+5 \le f(x) \le x^3+4x-7$, what is $\lim_{x\to 2} f(x)$?

(A) 2 (B) 5 (C) 9 (D) 11 (E) Not enough information

Squeeze thm

$\lim_{x\to 2} 2^x+5 = 4+5 = 9$

$\lim_{x\to 2} x^3+4x-7 = 8+8-7 = 9$

so, $\lim_{x\to 2} f(x) = 9$ also!

Part II—Free Response: Show all work in the space provided. Use proper notation.

Let a piecewise function be defined below.

$$f(x)=\begin{cases} \dfrac{2+e^x}{3-e^x}, & x<-8 \\ \dfrac{2x^3+10x^2}{|3x+15|}, & -8\le x\le -2 \\ x^2+3x-2, & -2<x<0 \\ -2, & x=0 \\ 2^x+1, & 0<x<1 \\ \sec x, & 1\le x<\dfrac{3\pi}{2} \\ \arctan x, & x\ge \dfrac{3\pi}{2} \end{cases}$$

(a) Using the 3-step definition of continuity at a point, determine if $f(x)$ is continuous at $x=0$.

$\lim_{x\to 0^-} f(x) = 0^2+3(0)-2=-2$

√1 need only one of these

$f(0)=-2$

$\lim_{x\to 0^+} f(x) = 2^0+1=1+1=2$ √2

Since $-2\neq 2$, √3

$f(x)$ is not continuous at $x=0$ √4

(b) $\lim_{x\to -5^-} f(x) =$

$\lim_{x\to -5^-} f(x) =$

$\lim_{x\to -5^-} \dfrac{2x^3+10x^2}{|3x+15|}$

$\dfrac{2(-5)^3+10(-5)^2}{|3(-5)+15|} = 0/0$

$\lim_{x\to -5^-} \dfrac{2x^2(x+5)}{|3x+15|}$ √5

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$\lim_{x\to -5^-}(2x^2)\cdot \lim_{x\to -5^-}\dfrac{x+5}{|3x+15|}$

$2(-5)^2\cdot\left(\dfrac{-6+5}{|3(-6)+15|}\right)$ plug in -6 to this piece only

$50\cdot\dfrac{-1}{3}$

$-\dfrac{50}{3}$ √6

(c) $\lim_{x \to -\infty} f(x) =$

$\lim_{x \to -\infty} f(x)$

$\lim_{x \to -\infty} \frac{2+e^x}{3-e^x} = \frac{2+e^{-\infty}}{3-e^{-\infty}}$

$\frac{2+0}{3-0}$

$\frac{2}{3}$ $\sqrt{7}$

(d) $\lim_{x \to \infty} f(x) =$

$\lim_{x \to \infty} f(x)$

$\lim_{x \to \infty} \arctan x$

$\frac{\pi}{2}$ $\sqrt{8}$

(e) $\lim_{x \to \frac{\pi}{3}} f(x) =$

$\lim_{x \to \frac{\pi}{3}} f(x)$

$\lim_{x \to \frac{\pi}{3}} \sec x$

$\sec\frac{\pi}{3}$ or 2 $\sqrt{9}$