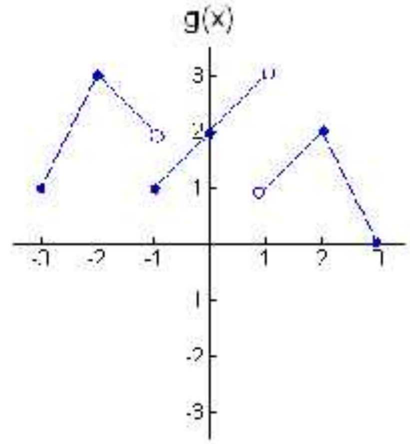
Name KEY Date________________ Period__________

AP Calculus TEST: 1.1-1.4 No Calculator

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.



Use the graph of the function $g(x)$ shown at right to answer question 1.

C 1. $\lim\limits_{x\to -3^+} g(g(x)) + \lim\limits_{x\to 1^-}\sqrt{3g(x)} + \lim\limits_{x\to -1^-} g^2(x) = =$

(A) 6 (B) 7 (C) 8 (D) DNE

$g(\lim\limits_{x\to -3^+} g(x)) + \sqrt{3\cdot 3} + [\lim\limits_{x\to -1^-} g(x)]^2$

$\lim\limits_{x\to 1^+} g(x) + \sqrt{9} \qquad 2^2$

$1 + 3 + 4$

8

D 2. $\lim\limits_{x\to 7}\dfrac{2x^2-13x-7}{x-7} =$ $0/0$

(A) ∞ (B) 0 (C) 2 (D) 15

$\lim\limits_{x\to 7}\dfrac{\cancel{(x-7)}(2x+1)}{\cancel{(x-7)}}$

$\lim\limits_{x\to 7}(2x+1)$

15

B 3. If $\csc\left(\dfrac{7\pi}{x}\right) \le P(x) \le \ln\left(\dfrac{12}{x}-1\right) - \sqrt{x-2}$, for all x in an interval containing $x=6$, then $\lim\limits_{x\to 6} P(x) =$

(A) -1 (B) -2 (C) $\dfrac{-2\sqrt{3}}{3}$ (D) not enough information is given

$\lim\limits_{x\to 6}\csc\left(\frac{7\pi}{x}\right)$, $\lim\limits_{x\to 6}\ln\left(\frac{12}{x}-1\right)-\sqrt{x-2}$ So $\lim\limits_{x\to 6} P(x) = -2$ also

$\csc\frac{7\pi}{6} \qquad \ln 1 - \sqrt{4}$

$-2 \qquad 0-2$

-2

B 4. $\lim\limits_{x\to -\infty}\dfrac{-3x^5+9x^4+1}{\sqrt{9x^{10}+x^8+4}} =$

effective growth rate (A) -1 (B) 1 (C) $-\dfrac{1}{3}$ (D) $\dfrac{1}{3}$

$\dfrac{-3x^5}{3x^5}$ → approaches some type of $\left|\frac{-3}{3}\right| = 1$

* check sign by plugging in a negative into original. → Positive so limit is +1

A____ 5. $\lim_{x \to 6^+} \frac{(x+5)(7-x)}{(x-5)(6-x)} =$ $\frac{\neq 0}{0} \to VA \to \pm\infty$

plug in $x = 6.1$ to check sign $\to$ neg, so $-\infty$

(A) $-\infty$ (B) ∞ (D) 1 (E) -1

B____ 6. If $f(x) = \begin{cases} ax^2 + 2b, & x < 1 \\ 2, & x = 1 \\ 3ax - 5b, & x > 1 \end{cases}$ is continuous at $x = 1$ for particular values of real numbers *a* and *b*, what is the value of $\frac{a}{b}$?

$\lim_{x \to 1^-} f(x) = a + 2b$

$\lim_{x \to 1^+} f(x) = 3a - 5b$

$f(1) = 2$

so $\begin{cases} a + 2b = 2 \to a = 2 - 2b \\ 3a - 5b = 2 \leftarrow \text{sub} \end{cases}$

$3(2 - 2b) - 5b = 2$

$6 - 6b - 5b = 2$

$-11b = -4$

$b = \frac{4}{11}$

$a = 2 - 2\left(\frac{4}{11}\right)$

$a = \frac{22 - 8}{11}$

$a = \frac{14}{11}$

$\frac{a}{b} = \frac{14}{11} \cdot \frac{11}{4}$

$= \frac{7}{2}$

(A) $\frac{14}{11}$ (B) $\frac{7}{2}$ (C) $\frac{4}{11}$ (D) no such values of *a* and *b* exist

A____ 7. If $f(x)$ is continuous for all *x* in the interval $[a, b]$, then at any point $c \in (a, b)$, which of the following must be true?

(A) $\lim_{x \to c} f(x) = f(c)$ (B) $f(c) = 0$ (C) $f(c) = f(b) - f(a)$ (D) $f(a) \le f(c) \le f(b)$

3-step Def of Continuity at a point

B____ 8. Evaluate $\lim_{x \to 4^-} \frac{\sqrt{x+5} - 3}{x - 4}$ $\frac{0}{0}$ $\cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$

(A) 1 (B) $\frac{1}{6}$ (C) $-\frac{1}{6}$ (D) -6

$\lim_{x \to 4^-} \frac{(x + 5 \cancel{-9})}{(\cancel{x-4})(\sqrt{x+5} + 3)}$ 1

$\frac{1}{3 + 3}$

$\frac{1}{6}$

Part II: Free Response: Answer all questions in the rectangle provided for each problem. **Show all steps, use proper notation, and write legibly.**

$$f(x)=\begin{cases} \dfrac{x^3-2x^2+3}{3x^2+4x-1}, & x<-3 \\ \dfrac{x^2+3x-1}{x-1}, & -3\le x<-2 \\ \dfrac{-2}{x}, & -2\le x<2 \\ 1-3x, & 2<x\le 4 \\ \dfrac{5+2^{-x}}{3-2^{-x}}, & x>4 \end{cases}$$

9. For the function given above, find the following.

(a) $\lim_{x\to 3} f(x) = 1-3(3) = -8$ $\sqrt{1}$

(b) $\lim_{x\to 0^-} f(x) =$ DNE or ∞ $\sqrt{2}$

(c) $\lim_{x\to \infty} f(x) = \frac{5}{3}$ $\sqrt{3}$

$$\frac{5+2^{-\infty}}{3-2^{-\infty}} = \frac{5+\frac{1}{2^{\infty}}}{3-\frac{1}{2^{\infty}}} = \frac{5+0}{3-0} = \frac{5}{3}$$

(d) $\lim_{x\to -\infty} f(x) =$ DNE or $-\infty$ $\sqrt{4}$

(e) Using the 3-step definition of continuity, discuss the continuity of $f(x)$ at $x = -2$.

$\lim_{x \to -2^-} f(x) = \frac{4-6-1}{-2-1} = \frac{-3}{-3} = 1$

$\lim_{x \to -2^+} f(x) = \frac{-2}{-2} = 1$

$f(-2) = 1$

$\sqrt{5}$

f(x) is continuous at $x = -2$ since $1 = 1 = 1$

$\sqrt{6}$

(f) Using the 3-step definition of continuity, discuss the continuity of $f(x)$ at $x = 2$.

$\lim_{x \to 2^-} f(x) = -1$

$\lim_{x \to 2^+} f(x) = -5$

-OR-

$f(2) = \text{DNE}$

$\sqrt{4}$

f(x) is NOT continuous at $x=2$ since $\begin{cases} -1 \neq -5 \\ \text{or} \\ f(2) = \text{DNE} \end{cases}$

$\sqrt{8}$