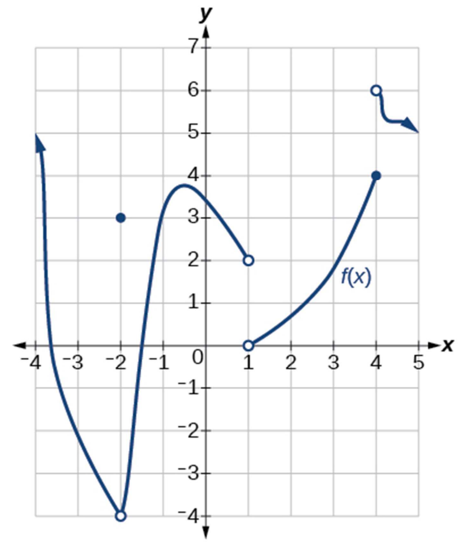
Name KEY Date________ Favorite Temperature________

AP Calculus TEST 1.1-1.3, No Calculator

Part I—Multiple Choice: Put the correct CAPTIAL LETTER in the space provided next to each question number.



D 1. Using the graph of $f(x)$ on the right, what is the value of

$$\lim_{x\to 4^-} f(x-3) + \lim_{x\to -2}\left[f(x)\right]^2 - \lim_{x\to 4^+} f(x)$$

(A) 5 (B) 7 (C) 10 (D) 12 (E) 26

$2 + 16 - 6 = 12$

C 2. Using the graph of $f(x)$ on the right, on the open interval $-4 < x < 5$, how many discontinuities does the graph of $f(x)$ have?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

E 3. $\lim_{x\to\infty} \dfrac{4x + 2x^2 - 3x^3}{5x^3 + x^2 - 11} =$ $-\frac{3}{5}$

(A) $\frac{4}{5}$ (B) $-\frac{4}{5}$ (C) 0 (D) $\frac{3}{5}$ (E) $-\frac{3}{5}$

D 4. $\lim_{x\to -\infty} \dfrac{8x^3 + 2x^2 - 14}{\sqrt{16x^6 + 11x^4 + 9}} =$ $\frac{-8}{4} = -2$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) -2 (E) 2

D 5. $\lim_{x\to 2^-} \dfrac{x-5}{x-2} =$ $\frac{-3}{0}$

$+\infty$ (A) 0 (B) 1 (C) $\frac{5}{2}$ (D) ∞ (E) $-\infty$

(plug in 1.9 to get sign of infinity)

A 6. If $f(x)=\begin{cases} ax+b, & x<-1 \\ -3, & x=-1 \\ 2ax^2+bx, & x>-1 \end{cases}$ is continuous at $x=-1$, what is the value of $a\cdot b$?

(A) 54 (B) -15 (C) 3 (D) -9 (E) 28

$\lim_{x\to-1^-} f(x) = -a+b$

$f(-1)=-3$

$\lim_{x\to-1^+} f(x) = 2a-b$

So $-a+b=-3$
$+\ 2a-b=-3$
$a=-6$
So, $6+b=-3$
$b=-9$
and $ab=(-6)(-9)=54$

A 7. $\displaystyle\lim_{x\to7}\frac{x^2-5x-14}{x^2-10x+21}=$ $\frac{0}{0}$

$\lim_{x\to7}\frac{\cancel{(x-7)}(x+2)}{\cancel{(x-7)}(x-3)}$

(A) $\frac{9}{4}$ (B) $-\frac{2}{3}$ (C) $-\frac{5}{7}$ (D) 1 (E) DNE

$\frac{7+2}{7-3}$

$\frac{9}{4}$

C 8. If $2^x+5\le f(x)\le x^3+4x-7$, what is $\lim_{x\to2} f(x)$?

(A) 2 (B) 5 (C) 9 (D) 11 (E) Not enough information

Squeeze Thm

$\lim_{x\to2}(2^x+5)=4+5=9$

$\lim_{x\to2}(x^3+4x-7)=8+8-7=9$

So, $\lim_{x\to2} f(x)=9$ also!

C 9. Which of the following is NOT an equation of an asymptote to the function

$$f(x)=\frac{x^3+3x^2-10x-30}{x^2-x-6}?$$

I. $x=3$
II. $x=-2$
~~III.~~ $y=x+3$
IV. $y=x+4$

$f(-2)=\frac{-8+12+20-30}{4+2-6}=\frac{-6}{0}$, so f has a VA @ $x=-2$

$f(3)=\frac{27+27-30+30}{9-3-6}=\frac{54}{0}$, so f has a VA @ $x=3$

(A) I and II only (B) I, II, and III only (C) I, II, and IV only (D) II and IV only (E) I and IV only

Long Division: for Slant Asymptote Equation.

$$\begin{array}{r} x+4 \\ x^2-x-6\,\big)\,\overline{x^3+3x^2-10x-30} \\ -x^3+x^2+6x \\ \hline 4x^2-4x \end{array}$$

So, f has an S.A. @ $y=x+4$

Part II—Free Response: Show all work in the space provided. Use proper notation.

Let a piecewise function be defined below.

$$f(x)=\begin{cases} \dfrac{2+e^x}{3-e^x}, & x<-8 \\ \sqrt{x+8}, & -8\le x\le -4 \\ x^2+3x-2, & -4<x<0 \\ -2, & x=0 \\ 2^x+1, & 0<x<1 \\ \sec x, & 1\le x<\dfrac{3\pi}{2} \\ \arctan x, & x>\dfrac{3\pi}{2} \end{cases}$$

(a) Using the 3-step definition of continuity at a point, determine if $f(x)$ is continuous at $x=-4$. $\lim_{x\to -4^-} f(x)=\sqrt{-4+8}=2$ $f(-4)=2$ ✓1 $\lim_{x\to -4^+} f(x)=(-4)^2+3(-4)-2=2$ Since $2=2=2$, ✓2 $f(x)$ is continuous @ $x=-4$ ✓3
(b) Using the 3-step definition of continuity at a point, determine if $f(x)$ is continuous at $x=0$. $\lim_{x\to 0^-} f(x)=-2$ ✓4 $f(0)=-2$ $\lim_{x\to 0^+} f(x)=2$ Since $-2\neq 2$, ✓5 $f(x)$ is not continuous @ $x=0$ ✓6

(c) $\lim_{x \to -\infty} f(x) =$ $\lim_{x \to -\infty} f(x)$ $\lim_{x \to -\infty} \frac{2+e^x}{3-e^x}$ $\frac{2+e^{-\infty}}{3-e^{-\infty}}$ $\frac{2+0}{3-0}$ $\frac{2}{3}$ √7
(d) $\lim_{x \to \infty} f(x) =$ $\lim_{x \to \infty} f(x)$ $\lim_{x \to \infty} \arctan x$ $\frac{\pi}{2}$ √8
(e) $\lim_{x \to \frac{\pi}{3}} f(x) =$ $\lim_{x \to \frac{\pi}{3}} f(x)$ $\lim_{x \to \frac{\pi}{3}} \sec x$ $\sec\frac{\pi}{3}$ or 2 √9