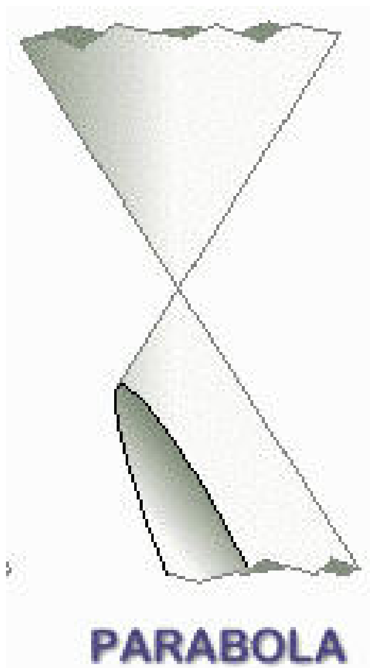
Déjà Vu, It's Algebra 2!

Lesson 31

Conic Sections continued: Parabolas

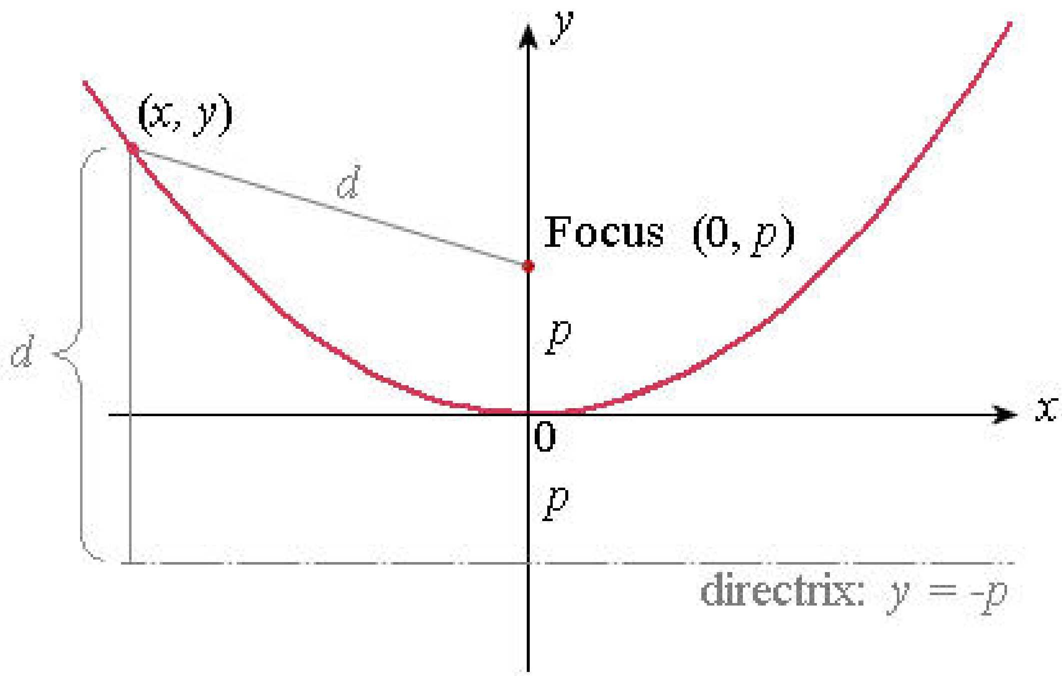
A **PARABOLA** is formed by slicing a cone at an angle that is slanted the same as, or parallel to, the cone.



The name parabola comes from the Greek roots, *para,* meaning "along side to" and *bola,* meaning "to throw." The word parallel has the same first root. The parabola has many uses, ranging from trajectories of objects to their use as reflectors in cases where objects must be concentrated at a single point.

Locus Definition of a Hyperbola:

The set of all points in the plane whose distances from a fixed point, called the *focus*, and a fixed line, called the *directrix*, are always equal.



The point directly between, and hence closest to, the focus and the directrix is called the *vertex* of the parabola.

To derive the equation of a parabola in rectangular coordinates, we again choose a convenient location for the axes, placing the origin at the vertex so that the *y*-axis is the axis of symmetry. We denote the distance from the vertex to the focus by *p*, so that the directrix is then the line $y=-p$.

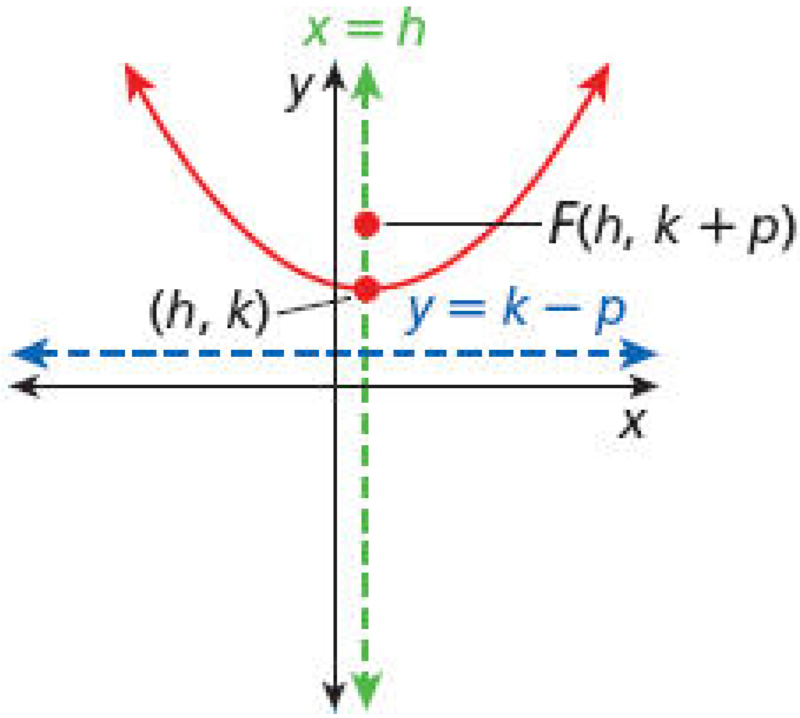
$$y+p=\sqrt{(x-0)^2+(y-p)^2}$$

Algebra reduces this to the standard equation of a parabola opening vertically.

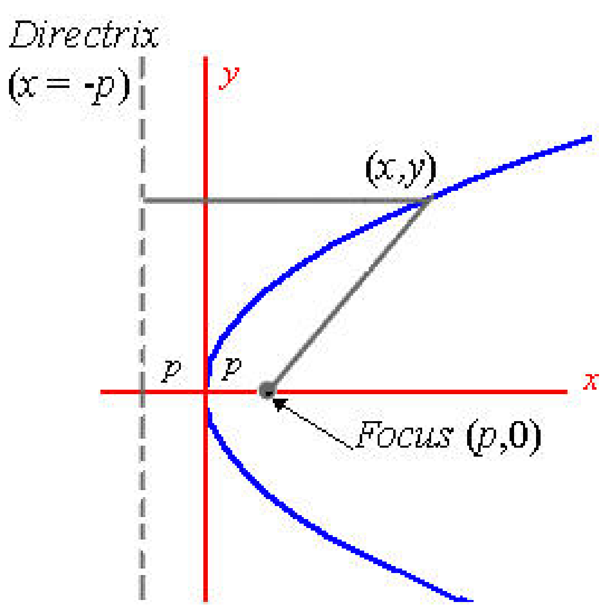
$$y=\frac{1}{4p}x^2$$

We can now modify this equation for one whose vertex is at (h,k)

Parabola $y - k = \frac{1}{4p}(x - h)^2$



If the parabola opens horizontally, it is NOT a quadratic function (but still a parabola.) It's standard equation would be:



$$x - h = \frac{1}{4p}(y - k)^2$$

If $p > 0$, the parabola opens in the positive direction, up or to the right.

If $p < 0$, the parabola opens in the negative direction, down or to the

Let's graph one ourselves.

Example:

Graph the following parabola. Show the vertex, focus, and the directrix. $x = \frac{1}{2}(y+1)^2 + 2$

We must first put it into standard form:

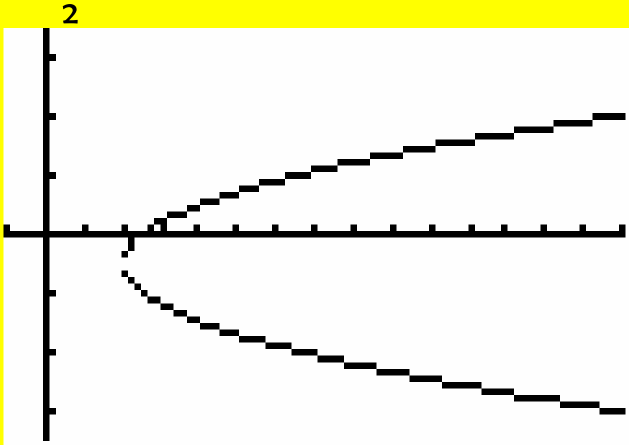
$x - 2 = \frac{1}{2}(y+1)^2$. This is a parabola opening horizontally, with a vertex at $(2,-1)$. Since the coefficient of the squared term is positive, it opens in the positive direction, to the right in this case. But what is *p*???

The coefficient of the squared term is always equal to $\frac{1}{4c}$.

$\frac{1}{4c} = \frac{1}{2}$

$2 = 4c$

$c = \frac{1}{2}$



Vertex at $(2,-1)$

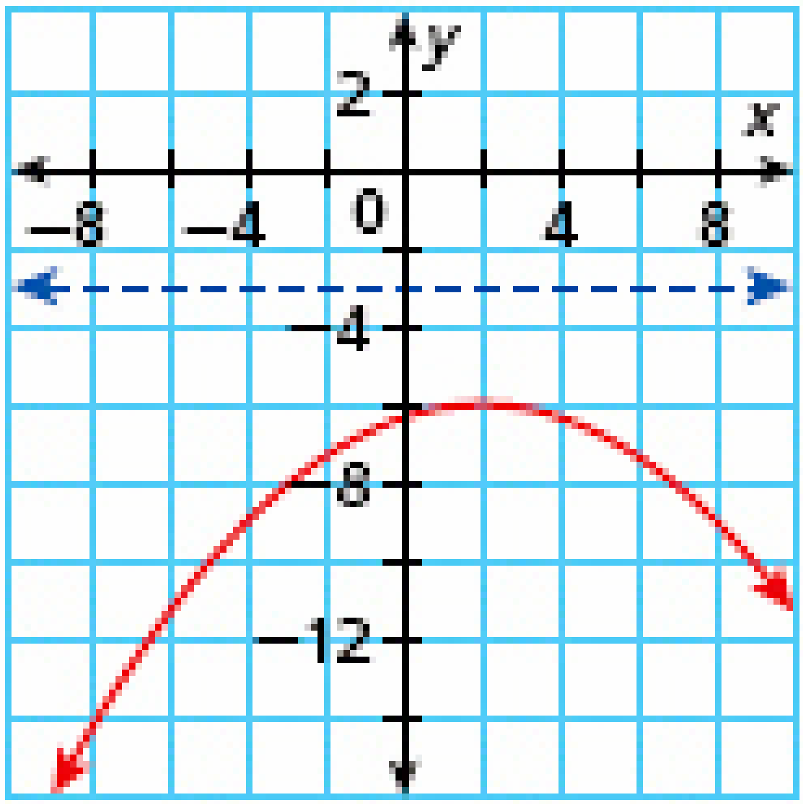
Focus at $(2.5,-1)$

Directrix at $x = 1.5$

We can also write equations of parabolas from given information.

Example:

Write an equation in standard form of the following parabola, then find the coordinate of the focus.



Notice the scale of the graph is two, not one. We know that since the parabola opens vertically, it will be of the form $y-k=\frac{1}{4c}(x-h)^2$. Since it opens down, we know the value of c will be negative. All we need are the actual values of *h, k* and *c*, then we can "drop" them into the above equation.

The vertex is at $(2,-6)$, and the distance from the vertex to the closest point on the directrix is 3 units. Since the vertex is exactly halfway between the directrix and focus, this is our value of c, so $c=-3$.

So the equation is $y+6=\frac{1}{4(-3)}(x-2)^2$ or $y+6=-\frac{1}{12}(x-2)^2$. From this, we know the focus must be at the coordinate $(2,-6-3)=(2,-9)$

Example:

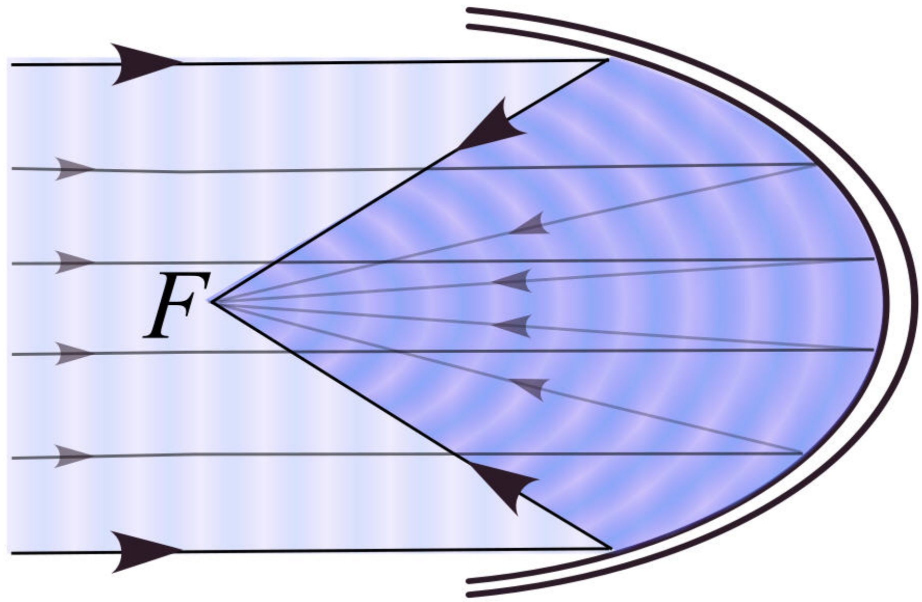
Write the equation in standard form of the parabola with focus at $(4,-5)$ and directrix at $x=12$.

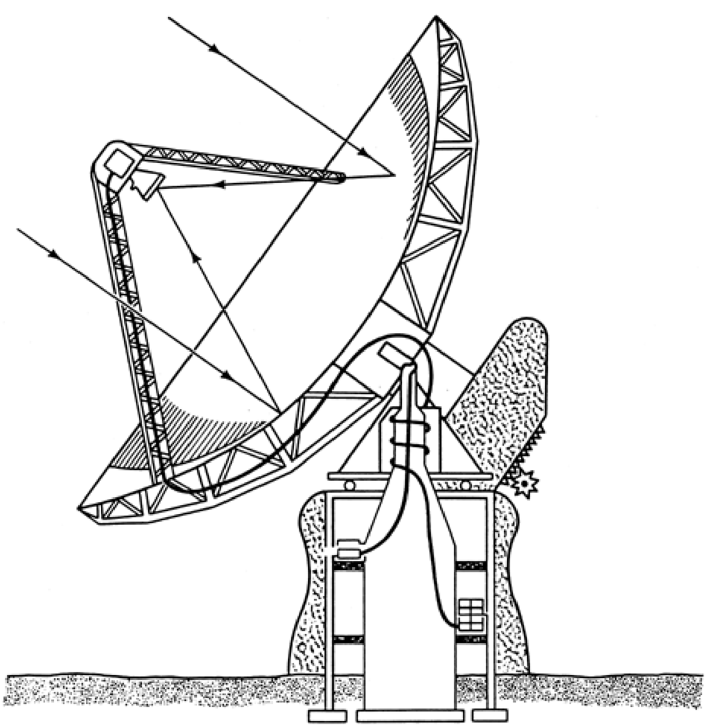
For a "blind" problem like this, it is important to draw the given information, which allows you to orient the parabola and to find your required information.
You'll see that this parabola opens to the LEFT. The vertex is halfway between the focus and directrix at $(8,-5)$ with $c=-4$.

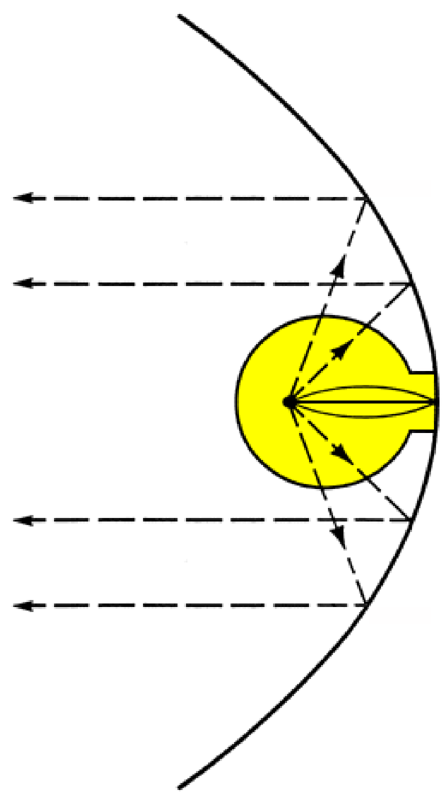
Therefore, the equation is $x-8=-\frac{1}{16}(y+5)^2$

Déjà RE-Vu

The reflective properties of parabola make it very useful in a variety of practical applications.







Let's say you are constructing a parabolic microphone. The surface the parabolic microphone will reflect sounds to the focus of the microphone at the end of a part called a feedhorn. The equation for the cross section of the parabolic microphone dish is $x = \frac{1}{32}y^2$, measured in inches.

How long should you make the feedhorn?

Because we are measuring relative position, the vertex of this cross section is at the origin and opens to the right for convenience. We want to place the microphone at the end of the feedhorn precisely at the focus of the parabola. From the equation, we want to find the value of *c*.

$$\frac{1}{4c} = \frac{1}{32}$$

$$4c = 32$$

$$c = 8 \text{ inches}$$

Therefore, the feedhorn should be 8 inches long.

Math is everywhere!

References:

http://id.mind.net/~zona/mmts/miscellaneousMath/conicSections/parabola.htm
http://www.algebra.com/algebra/homework/Quadratic-relations-and-conic-sections/Parabola.wikipedia
http://go.hrw.com
http://www.mathacademy.com/pr/prime/articles/conics/index.asp
http://www.intmath.com/Plane-analytic-geometry/4_Parabola.php
http://www.answers.com/topic/parabola?cat=technology
http://en.wikipedia.org
http://britton.disted.camosun.bc.ca/jbconics.htm
http://sol.sci.uop.edu/~jfalward/reflection/reflection.html