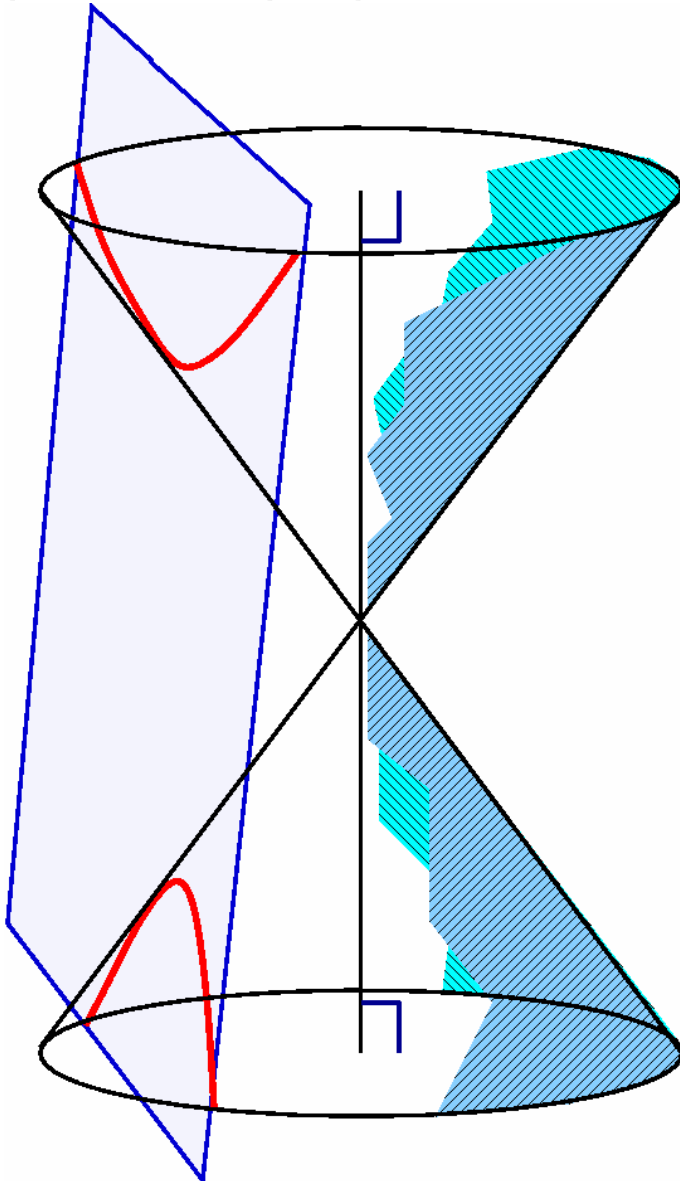# *Déjà Vu, It's Algebra 2!*
Lesson 30
Conic Sections continued: Hyperbolas

A **HYPERBOLA** is formed by slicing a double-knapped cone perpendicular to the base.

The hyperbola is the least known and used of the conic sections. We seldom see a hyperbola in daily life, and it seldom appears in decoration or design. In spite of this, it has interesting properties and important applications. There is a literary term, *hyperbole*, that is the same word in Greek, meaning an excess. Apollonius was the first to study the two branches of a hyperbola and gave it its name.

The Hyperbola has many applications: the path of a comet often takes the shape of a hyperbola, and many telescopes use hyperbolic (hyperbola-shaped) lenses. Hyperbolic gears are used in many machines, and in industry. Sound waves travel in hyperbolic paths, and so there are applications of the hyperbola in navigation.
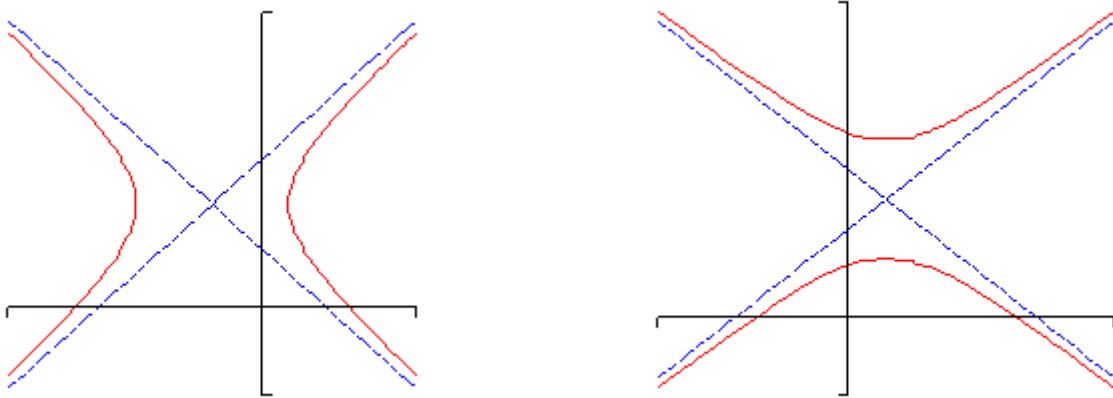
Locus Definition of a Hyperbola:

The set of all points whose DIFFERENCE of the distances to two fixed points, called the foci, is a constant.
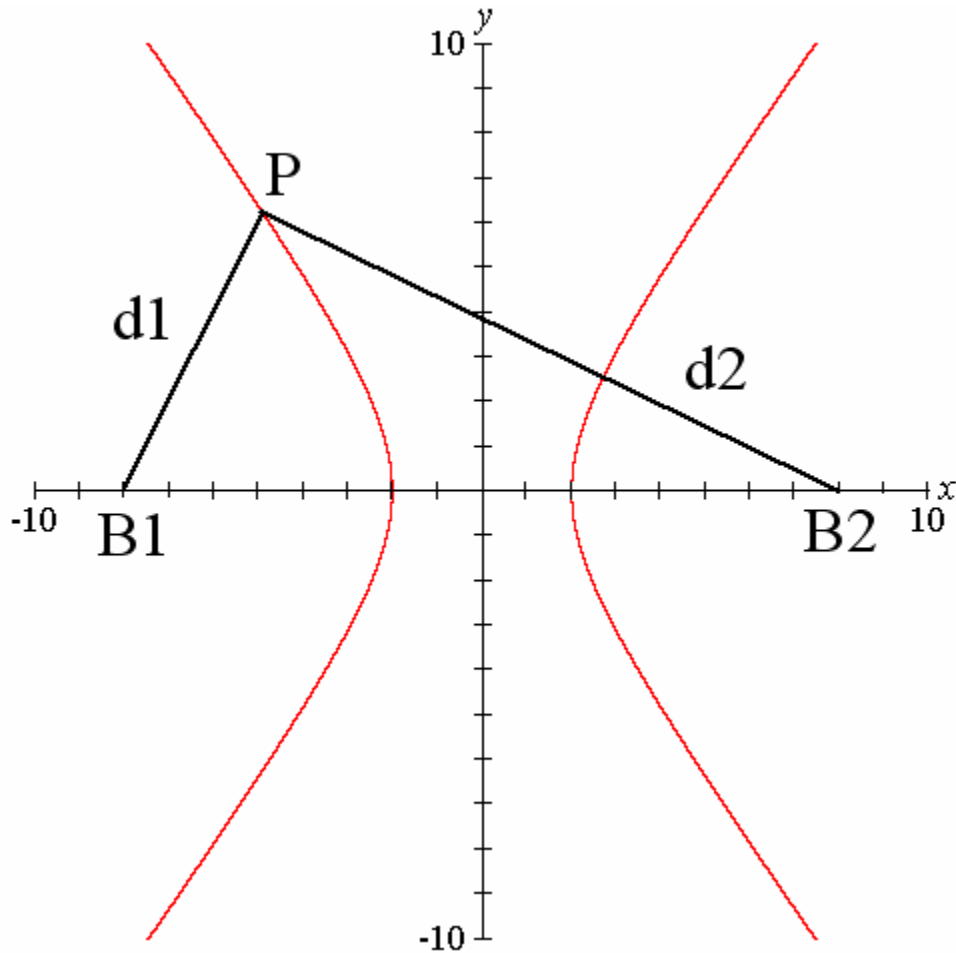
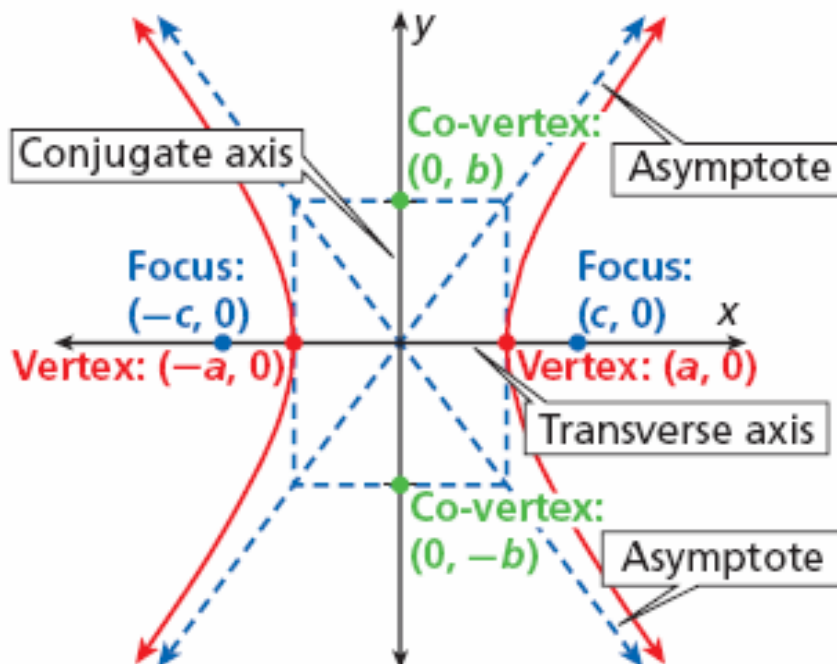$$\left| d_1 - d_2 \right| = \text{CONSTANT}$$

There are two basic varieties of hyperbola graphs:

There are two **standard forms** of the hyperbola, one for each type shown above. Here is a diagram of the horizontal variety as well as the information we can get from each one.

Hyperbola $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

Hyperbolas consist of two vaguely parabola shaped pieces that open either up and down or right and left. Also, just like parabolas each of the pieces has a vertex. Note that they aren't really parabolas, they just resemble parabolas.

There are also two lines on each graph. These lines are called asymptotes and as the graphs show as we make x large (in both the positive and negative sense) the graph of the hyperbola gets closer and closer to the asymptotes. The asymptotes are not officially part of the graph of the hyperbola. However, they are usually included so that we can make sure and get the sketch correct. The point where the two asymptotes cross is called the center of the hyperbola.

A hyperbola that opens vertically will be of the form:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Note that the difference between the two forms is which term has the minus sign. If the y term has the minus sign then the hyperbola will open left and right. If the x term has the minus sign then the hyperbola will open up and down. That is, the hyperbola will always cross the axis of the POSITIVE term!!
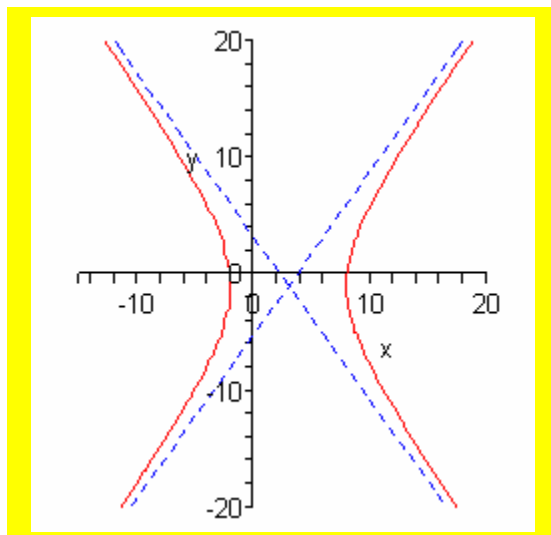
The special relation among the variables a, b, and c is:
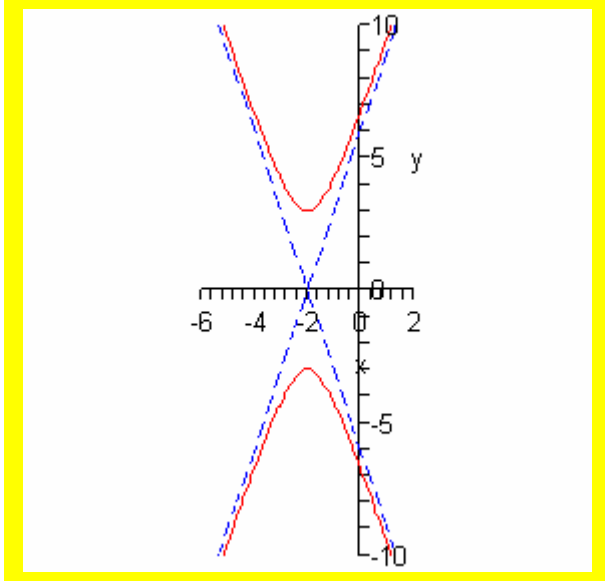
$$c^2 = a^2 + b^2$$

Let's try to graph one. . .

Example:

$$\frac{(x-3)^2}{25} - \frac{(y+1)}{49} = 1$$

Example:

Graph $\dfrac{y^2}{9} - (x+2)^2 = 1$

Aside from being able to graph hyperbolas given the equation, it is important to be able to write equations from a graph or given information.

Example:
Find the equation of a hyperbola with center (1, 1), vertex (3, 1) and focus at (5, 1).

$$\frac{(x-1)^2}{4} - \frac{(y-1)}{12} = 1$$

Déjà RE-Vu

Reflective Property of a Hyperbola:
Like other conics, a hyperbola can be made into an excellent reflector of sound, light, and other waves.

<u>Long-Range Navigation:</u>
Hyperbolas and radio signals are the basis of the LORAN (long-range navigation) system.

Developed in the US in 1940, LORAN works by comparing time differences between radio transmitters. Radio transmitters are setup in a chain of three or more are separated by hundreds of miles. A chain will have one master transmitter and a series of secondary transmitters. The stations constantly transmit signals with precise timing information.
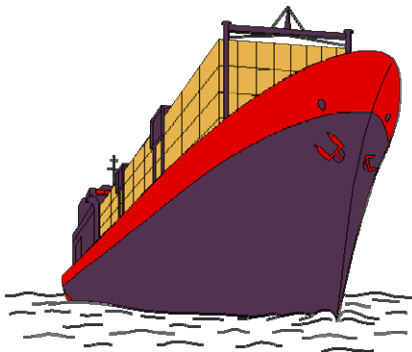
A LORAN receiver compares the difference between the timing signals from the master-secondary transmitters and measures the difference.   With this information, your position somewhere on a curved line from the transmitter can be determined. To find out where you are on the curved line, a second signal from another chain is

required.   The second chain gives another curved line indicating your position on a curved line.

 Where the curved lines intersect from each of the chains is your exact position.
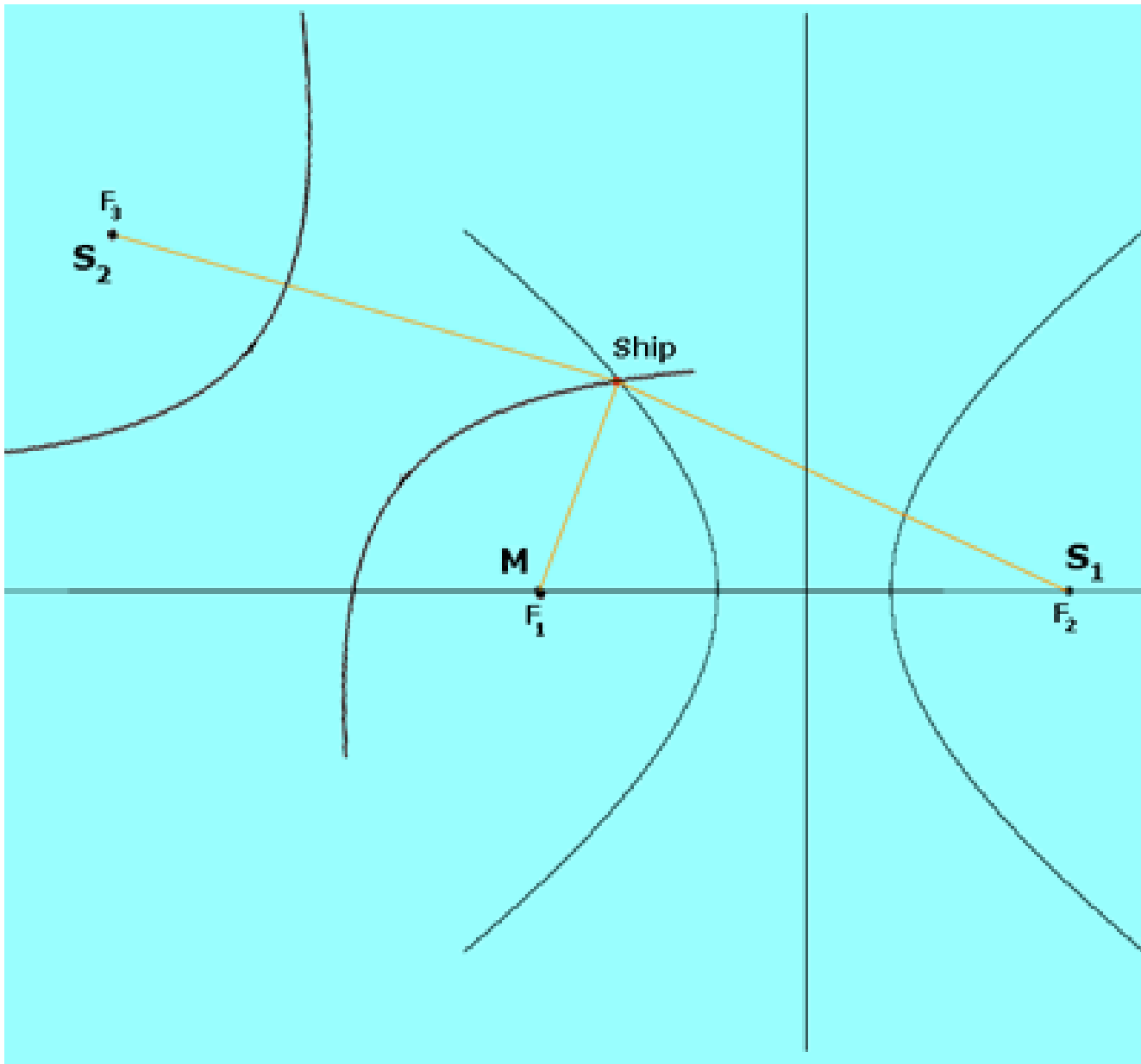
Math is everywhere!

References:

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