Name_____ Date_____ Period_____

Worksheet 4.3—The Fundamental Theorem of Calculus
Show all work. No calculator unless otherwise stated.

Multiple Choice

1. (Calculator Permitted) What is the average value of $f(x)=\cos x$ on the interval $[1,5]$?

(A) –0.990 (B) –0.450 (C) –0.128 (D) 0.412 (E) 0.998

2. If the average value of the function f on the interval $[a,b]$ is 10, then $\int_a^b f(x)\,dx =$

(A) $\frac{10}{b-a}$ (B) $\frac{f(a)+f(b)}{10}$ (C) $10b-10a$ (D) $\frac{b-a}{10}$ (E) $\frac{f(a)+f(b)}{20}$

3. (Calculator Permitted) Let $f'(x)=\ln(2+\sin x)$. If $f(3)=4$, then $f(5)=$

(A) 0.040 (B) 0.272 (C) 0.961 (D) 4.555 (E) 6.667

4. What is $\lim_{h\to 0}\frac{1}{h}\int_{x}^{x+h} f(t)\,dt$?

(A) 0 (B) 1 (C) $f'(x)$ (D) $f(x)$ (E) nonexistent

5. What is the linearization of $f(x)=\int_{\pi}^{x}\cos^3 t\,dt$ at $x=\pi$?

(A) $y=-1$ (B) $y=-x$ (C) $y=\pi$ (D) $y=x-\pi$ (E) $y=\pi-x$

6. (Calculator Permitted) The area of the region enclosed between the graph of $y=\sqrt{1-x^4}$ and the x-axis is

(A) 0.886 (B) 1.253 (C) 1.414 (D) 1.571 (E) 1.748

Short Answer

7. Let *f* be a function such that $f''(x) = 6x + 12$.

(a) Find $f(x)$ if the graph of *f* is tangent to the line $4x - y = 5$ at the point $(0, -5)$

(b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

8. Suppose *f* has a negative derivative for all values of *x* and that $f(1) = 0$. Which of the following statements must be true of the function

$$h(x) = \int_0^x f(t)\,dt\ ?$$

Give reasons for your answers.

(a) *h* is a twice-differentiable function of *x*.

(b) *h* and dh/dx are both continuous.

(c) The graph of *h* has a horizontal tangent at $x = 1$.

(d) *h* has a local maximum at $x = 1$.

(e) *h* has a local minimum at $x = 1$.

(f) The graph of *h* has an inflection point at $x = 1$.

(g) The graph of dh/dx crosses the *x*-axis at $x = 1$.

9. Find $\frac{dy}{dx}$

(a) $y = \int_{-\pi}^{x} \frac{2-\sin t}{3+\cos t}\,dt$

(b) $y = \int_{x}^{7} \sqrt{2m^4 + m + 1}\,dm$

(c) $y = \int_{x^3}^{5} \frac{\cos t}{t^2+1}\,dt$

(d) $y = \int_{\sqrt{x}}^{x^3} \sqrt{u}\sin u\,du$

10. If $F(x) = \int_{1}^{x} f(t)\,dt$, where $f(t) = \int_{1}^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, find $F''(2)$.

11. (Calculator Permitted) If $\frac{dy}{dx} = \sin^3 x$ and $y = 4$ when $x = 5$, construct and evaluate an integral equation to find

(a) $y(7)$ (b) $y(0)$ (c) $y(-2)$ d) $y(x)$

12. Evaluate without a calculator, then verify using fnINT(

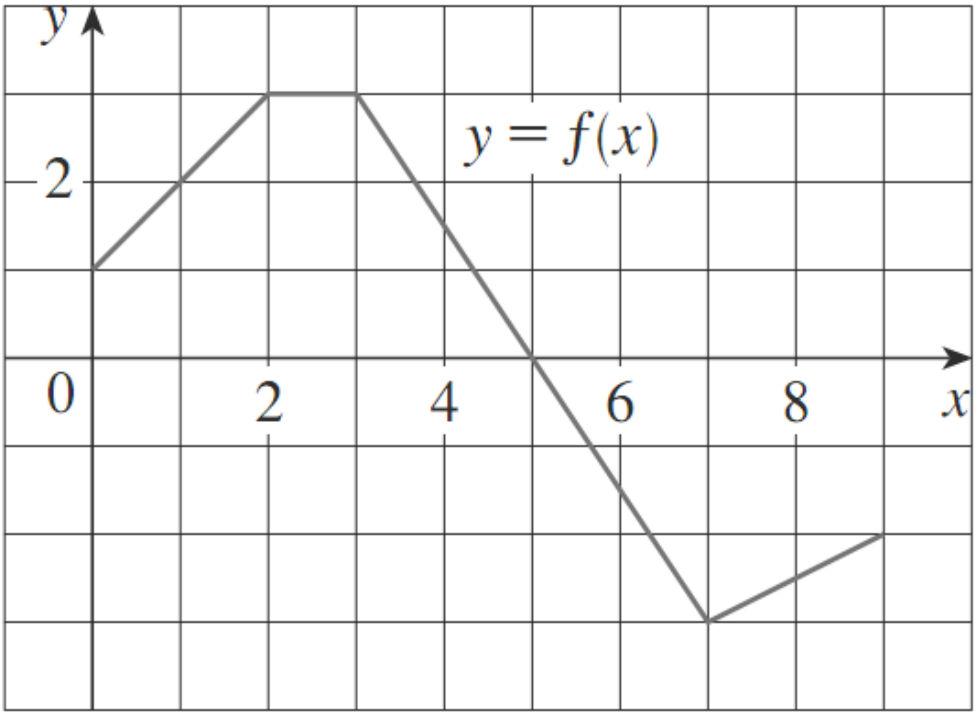
(a) $\int_2^{-1} 3^x \, dx$ (b) $\int_{-2}^{-1} \frac{1}{x^2} \, dx$ (c) $\int_0^1 \left(x^2 + \sqrt{x}\right) dx$ (d) $\int_{\pi/6}^{5\pi/6} \csc^2 \theta \, d\theta$ (e) $\int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}} \, du$

(f) $\int_0^2 x\left(2 + x^5\right) dx$ (g) $\int_0^1 \frac{4}{t^2+1} \, dt$ (h) $\int_0^2 f(x) \, dx$ where $f(x) = \begin{cases} x^4, & 0 \le x < 1 \\ x^5, & 1 \le x \le 2 \end{cases}$

13. Find the area of the region bounded by the *x*-axis and the curve $y = x^3 - 4x$ on $-2 \le x \le 2$

14. If $f(1) = 12$, $f'(x)$ is continuous, and $\int_1^4 f'(x) = 17$, what is the value of $f(4)$?

15. Find the average value of the following function on the given intervals. Verify with fnINT(
    (a) $f(x) = \cos x$ on $[0, \pi/2]$ (b) $f(x) = 1/x$ on $[1, 4]$ (c) $y = \sec x \tan x$ on $[0, \pi/4]$



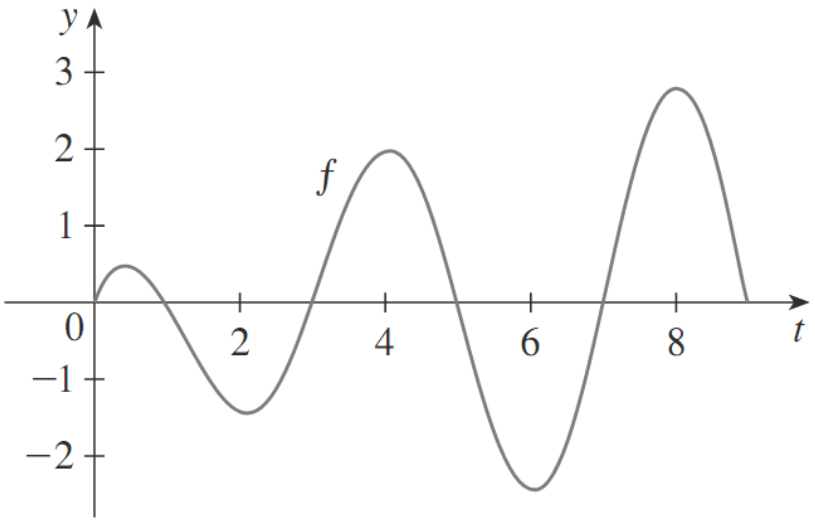
16. The graph of f is shown above. If $F(x)=\int_2^x f(t)\,dt$, evaluate the following using areas to help you.

(a) $F(0)$ (b) $F(2)$ (c) $F(5)$ (d) $F(7)-F(5)$ (e) $F(9)$

(f) where does F have a maximum value? A minimum value?

(g) What is the average value of $f(x)$ on $[2,9]$?

17. Let $g(x)=\int_0^x f(t)\,dt$, where f is the function whose graph is given below.



(a) At what values of x do the local maximum and local minimum of g occur? Justify.

(b) Where does g attain its absolute maximum value?

(c) On what approximate intervals is g concave downward?

(d) Sketch the graph of g.

18. (Calculator Permitted) If a cup of coffee has temperature $95^\circ C$ in a room where the temperature is $20^\circ C$, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is $T(t) = 20 + 75e^{-t/50}$.

What is the average temperature of the coffee during the first half hour? Show your integral set up. Include units in your final answer.