Name_____ Date_____ Period_____

Worksheet 3.4—Concavity and the Second Derivative Test

Show all work. No calculator unless otherwise stated.

Multiple Choice

_____ 1. If $a < 0$, the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on which of the following intervals?

(A) $\left(-\infty, -\frac{1}{a}\right)$ (B) $\left(-\infty, \frac{1}{a}\right)$ (C) $\left(-\frac{1}{a}, \infty\right)$ (D) $\left(\frac{1}{a}, \infty\right)$ (E) $(-\infty, -1)$

_____ 2. If $f(0) = f'(0) = f''(0) = 0$, which of the following **must** be true about the graph of f?

(A) There is a local max at the origin (B) There is a local min at the origin
(C) There is no local extremum at the origin (D) There is a point of inflection at the origin
(E) There is a horizontal tangent at the origin

_____ 3. The x-coordinates of the points of inflection of the graph of $y = x^5 - 5x^4 + 3x + 7$ are

(A) 0 only (B) 1 only (C) 3 only (D) 0 and 3 (E) 0 and 1

_____ 4. Which of the following conditions would enable you to conclude that the graph of f has a point of inflection at $x = c$?

(A) There is a local max of f' at $x = c$ (B) $f''(c) = 0$ (C) $f''(c)$ does not exist
(D) The sign of f' changes at $x = c$ (E) f is a cubic polynomial and $c = 0$

_____ 5. Let f be a twice-differentiable function on $(-\infty,\infty)$ such that the equation $f(x)=0$ has only and exactly 3 real solutions, all distinct, single root solutions. Consider the following possibilities:

I. the equation $f'(x)=0$ has exactly 2 distinct roots.

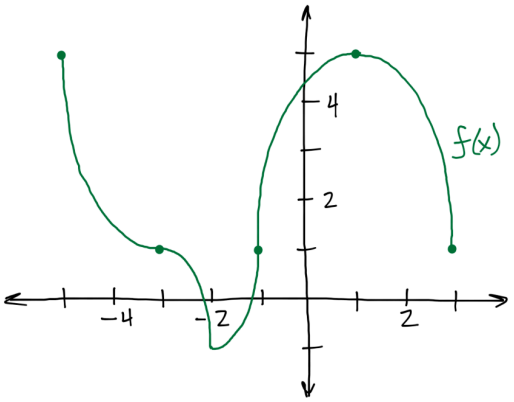
II. the equation $f''(x)=0$ has at least 1 root.

III. the function $f(x)$ must be a polynomial function of degree 3.

Which of these properties must f exhibit?

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 6. Let f be a continuous function on $[-5,3]$ with a vertical tangent line at $x=-1$, horizontal tangents at $x=-3$ and $x=1$ and a cusp at $x=-2$. The graph of f is given at right. On the interval $(-5,3)$, which of the following properties are satisfied?



I. $f''(x)<0$ on $(-2,1)$

II. f has exactly 2 local extrema

III. f has exactly 4 critical points

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

_____ 7.

x	0	2	4	6
$f''(x)$	2	6	0	-2

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(2,6)$.

(B) f is decreasing on the interval $(2,6)$.

(C) f has a local maximum at $x=4$.

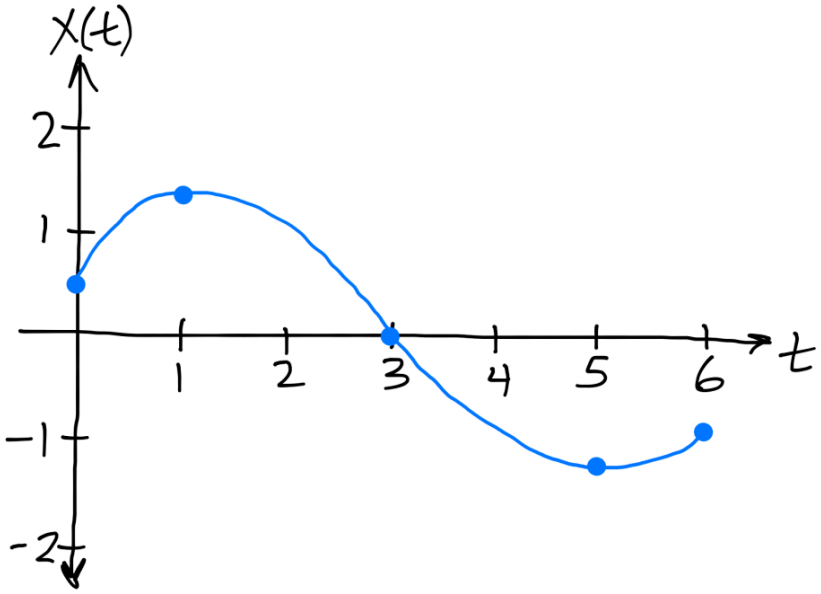
(D) The graph of f has a point of inflection at $x=4$.

(E) The graph of f changes concavity in the interval $(2,6)$.

_____ 8. Let f be a function with a second derivative given by $f''=x^2(x-4)(x-7)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) 4 only (C) 0 and 7 only (D) 4 and 7 only (E) 0, 4, and 7

_____ 9.



A particle moves along a straight line. The graph of the particle's posittion $x(t)$ at time t is shown above for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=3$. For what values of t is the velocity of the particle increasing?

(A) $0<t<3$

(B) $1<t<5$

(C) $3<t<6$

(D) $3<t<5$ only

(E) $0<t<1$ and $5<t<6$

_____ 10. (**Calculator Permitted**) The derivative of the function f is given by $f'(x)=x^2\sin\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2,2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

_____ 11. The function f is continuous on the closed interval $[3,5]$ and twice differentiable on the open interval $(3,5)$. If $f'(4)=2$ and $f''(x)<0$ on the open interval $(3,5)$, which of the followign could be a table of values for f?

(A)

x	$f(x)$
3	2.5
4	5
5	6.5

(B)

x	$f(x)$
3	2.5
4	5
5	7

(C)

x	$f(x)$
3	3
4	5
5	6.5

(D)

x	$f(x)$
3	3
4	5
5	7

(E)

x	$f(x)$
3	3.5
4	5
5	7.5

_____ 12. For what values of x does the graph of $y=x^5+5x^4+11$ have a point of inflection?

(A) $x=-4$ only

(B) $x=-3$ only

(C) $x=0$ only

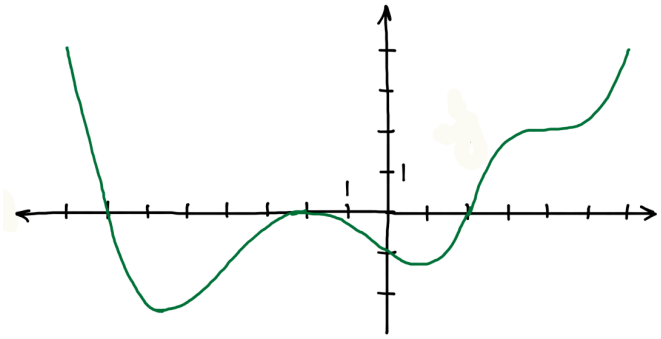
(D) $x=0$ and $x=-4$

(E) $x=0$ and $x=-3$

_____ 13. Let f be the function defined by $f(x)=2x^3-3x^2-12x+18$. On which of the following intervals is the graph of f both increasing and concave down?

(A) $(-\infty,-1)$ (B) $\left(-1,\frac{1}{2}\right)$ (C) $(-1,2)$ (D) $\left(\frac{1}{2},2\right)$ (E) $(2,\infty)$

_____14.



The graph of $f'(x)$

The figure above shows the graph of f', the derivative of function f, for $-8 < x < 6$. Of the following, which best describes the graph of f on the same interval?

(A) 1 local minimum, 1 local maximum, and 3 inflection points

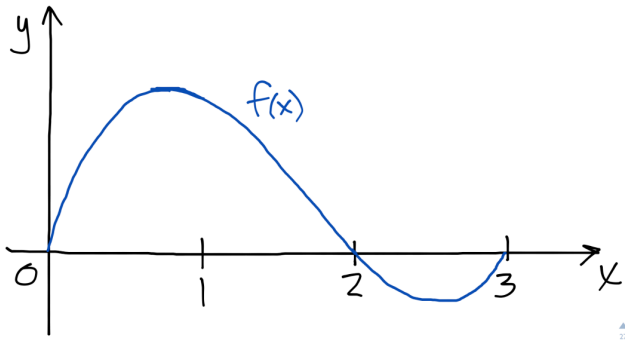
(B) 1 local minimum, 1 local maximum, and 4 inflection points

(C) 2 local minima, 1 local maximums, and 2 inflection points

(D) 2 local minima, 1 local maximum, and 4 inflection points

(E) 2 local minima, 2 local maxima, and 3 inflection points

_____ 15.



The graph of a differentiable function $f(x)$ is shown in the figure above and has an inflection point at $x = \frac{3}{2}$. Which of the following correctly orders $f(2)$, $f'(2)$, and $f''(2)$?

(A) $f(2) < f'(2) < f''(2)$

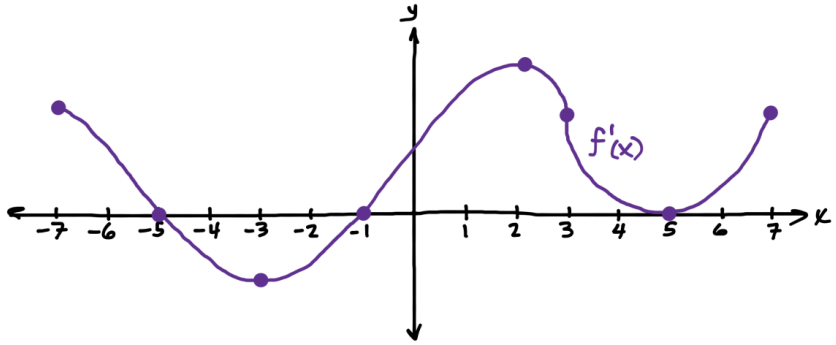
(B) $f'(2) < f(2) < f''(2)$

(C) $f'(2) < f''(2) < f(2)$

(D) $f''(2) < f(2) < f'(2)$

(E) $f''(2) < f'(2) < f(2)$

Short Answer



16. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all the values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all the values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all the open intervals, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) Find all the values of x, for $-7 < x < 7$, at which $f(x)$ has inflection points. Justify your answers.

(e) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

17. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$, $\forall x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(e) Does the **secant** line to the graph of h for $3 \le x \le 5$ lie above or below the graph of h for on $3 \le x \le 5$? Why?

18. A cubic polynomial function *f* is defined by $f(x) = 4x^3 + ax^2 + bx + k$, where *a*, *b*, and *k* are constants. The function *f* has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$. Find the values of *a* and *b*. (Hint: create ordered pairs satisfying $f(x)$, $f'(x)$, and $f''(x)$, then solve the system of equations).

19. For each of the following, (i) identify the open intervals on which the functions are concave up or concave down, then (ii) locate and **JUSTIFY** the inflection points)

(a) $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$

(b) $f(x) = \sin^{-1} x$, on $[-1,1]$

(c) $f(x) = \cos^2 x - 2\sin x,\ 0 \le x \le 2\pi$

(d) $f(x) = \dfrac{\ln x}{\sqrt{x}}$

20. Use the Second Derivative Test (if possible) to locate and **justify** the local extrema of the following functions. If the Test Fails, justify using the First Derivative Test.

(a) $f(x)=x^3-3x^2$

(b) $f(x)=3x^5-20x^3$

(c) $f(x)=2x^2\ln x$

(d) $f(x)=e^{-x}(x-7)$

21. Coffee is being poured into Mr. Korpi's mug (shown below) at a constant rate (measured in ounces per second). If the cup is filled to the brim, sketch a rough graph of the **depth** of the coffee in the mug, in inches, as a function of **time,** in seconds. Be sure to account for the shape of the graph, especially in terms of its concavity. What is the significance of the inflection point?